Lesson 4-4

Scientific Notation

Interactive Study Guide

See pages 79–80 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

Why is it useful to write numbers in different ways?

Common Core State Standards

Content Standards
8.EE.1, 8.EE.3, 8.EE.4

Mathematical Practices
1, 3, 4, 7

Vocabulary

standard form

scientific notation

What You'll Learn

- Express numbers in standard form and in scientific notation.
- Compare and order numbers written in scientific notation.

Real-World Link

Space Earth is the third planet from the Sun in our solar system. Because Earth's rotation about the Sun is not circular, the maximum distance between Earth and the Sun is about 95 million miles and the minimum distance is about 91 million miles.

Image by Reto Stöckli, NASA/Goddard Space Flight Center. Enhancements by Robert Simmon.

Key Concept **Scientific Notation**

Words	A number is expressed in scientific notation when it is written as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.
Symbols	$a \times 10^n$, where $1 \le a < 10$ and n is an integer.
Examples	$3{,}500{,}000 = 3.5 \times 10^6$ $\quad$ $0.00004 = 4 \times 10^{-5}$

Numbers that do not contain exponents are written in **standard form**. However, when you deal with very large numbers like 12,760,000 or very small numbers like 0.00001276, it can be difficult to keep track of the place value. A number that is expressed as a product of a factor and a power of 10 is written in **scientific notation**.

When a number is expressed in scientific notation the exponent tells you how many places to move the decimal point.

Example 1

Tutor

Express each number in standard form.

a. 2×10^3
$2 \times 10^3 = 2000$ — Move the decimal point 3 places to the right.

b. 6.8×10^5
$6.8 \times 10^5 = 680{,}000$ — Move the decimal point 5 places to the right.

c. 3.25×10^{-4}
$3.25 \times 10^{-4} = 0.000325$ — Move the decimal point 4 places to the left.

Got It? Do these problems to find out.

1a. 4×10^2 **400** $\quad$ **1b.** 5.94×10^7 **59,400,000** $\quad$ **1c.** 1.3×10^{-3} **0.0013**

2 Teach the Concept

Objectives write, compare, and order numbers in scientific notation

Building on the Essential Question

At the end of the lesson, students should be able to answer "Why is the conclusion that 7.8×10^3 is greater than 6.5×10^2 because $7.8 > 6.5$ incorrect?"

Example 1

What's the Math? write numbers in standard form

- *Given a number expressed in scientific notation, what does a positive exponent tell you?* **It tells you how many places to move the decimal point to the right when you write the number in standard form.**
- *Given a number expressed in scientific notation, what does a negative exponent tell you?* **It tells you how many places to move the decimal point to the left when you write the number in standard form.**

Need Another Example?

Express each number in standard form.

a. 3×10^5 **300,000**

b. 4.395×10^4 **43,950**

c. 6.79×10^{-6} **0.00000679**

Example 2

What's the Math? write numbers in scientific notation

- *When you express a number in scientific notation, how do you determine the value of the exponent?* **Count the number of places to the right of the leftmost digit if the number is greater than 1, and from the right of the decimal point to the first nonzero digit, if the number is less than 1.**

Need Another Example?

Express each number in scientific notation.

a. 800,000 8×10^5 **b.** 0.0119 1.19×10^{-2}

Example 3

What's the Math? estimate numbers in scientific notation

- *What is the first step in estimating with scientific notation?* **Round the number in standard form to the greatest place value.**

Need Another Example?

The population of Montana is 998,199. Write an estimation in scientific notation for the population.
Sample answer: 1×10^6

Example 4

What's the Math? choose units of appropriate size

- *If you walked 1 mile, would you say that you walked for about 20 minutes or about 1200 seconds?* **20 min**

Need Another Example?

If you could ride your bike 200 meters per second, it would take about 1.92×10^6 seconds to ride to the moon. Is it more appropriate to report this time as about 1.92×10^6 seconds or about 22.2 days?
22.2 days; The number of meters to the moon is very large, so the larger unit is more appropriate.

When expressing a number in scientific notation, the sign of the exponent can be determined by evaluating the number in standard form. If a number in standard form is greater than or equal to 1, then the exponent is *positive*. If a number is between 0 and 1, then the exponent is *negative*.

Scientific Notation
When numbers are expressed in scientific notation, no more than one digit is to the left of the decimal point.

Example 2

Tutor

Express each number in scientific notation.

a. 4,000,000
$4{,}000{,}000 = 4 \times 10^6$ — The decimal point moves 6 places. The exponent is positive.

b. 0.072
$0.072 = 7.2 \times 10^{-2}$ — The decimal point moves 2 places. The exponent is negative.

Got It? Do these problems to find out.

2a. 900 9×10^2 **2b.** 18,900 1.89×10^4 **2c.** 0.000064 6.4×10^{-5}

One way to estimate a very large or a very small number is to express it in the form of a single digit times an integer power of 10. For example, the population of the United States in 2010 was 308,745,538. The number 3×10^8 is an estimate of that number.

Example 3

Tutor

The population of Kansas is 2,853,118 people. Write an estimation in scientific notation for the population.

$2{,}853{,}118 \approx 3{,}000{,}000$ — Estimate.
$3{,}000{,}000 = 3 \times 10^6$ — Write in scientific notation.

The population of Kansas is about 3×10^6 people.

Got It? Do these problems to find out.

Estimate each value using scientific notation. **3a–3c. Sample answers are given.**

3a. 3,612,500 cm **4×10^6 cm**
3b. 0.000000251 ft **3×10^{-7} ft**
3c. 4.215×10^{-3} kg **4×10^{-3} kg**

Real World

Example 4

Tutor

STEM **The space shuttle traveled at about 8 kilometers per second. At this rate, the shuttle would take about 4.5×10^4 seconds to fly to the moon. Is it more appropriate for a newspaper to report this time as about 4.5×10^4 seconds or about 12.5 hours? Explain your reasoning.**

The measure 12.5 hours is more appropriate. The number 4.5×10^4 seconds is very large, so choosing the larger unit of measure is more meaningful.

Got It? Do this problem to find out.

4. A dime is about 5.875×10^{-3} foot in diameter. Is it more appropriate to report that the diameter of a dime is 5.875×10^{-3} foot or 7.05×10^{-1} inch? Explain your reasoning.

4. 7.05×10^{-1} in.; A dime's diameter is not very large, so choosing the smaller unit of measure is more meaningful.

Compare and Order Numbers

To compare and order numbers in scientific notation, first compare the exponents. With positive numbers, the number with a greater exponent is greater. If the exponents are the same, compare the factors.

Power of 10
When writing a number in scientific notation, the power of 10 is determined by the direction and number of places you move the decimal point.

Example 5

STEM The table shows different geologic time periods. Order the time periods from oldest to youngest.

Geologic Time Periods	
Period	**Number of Years Ago**
Jurassic	2.08×10^8
Silurian	4.38×10^8
Tertiary	6.64×10^7
Triassic	2.45×10^8

Step 1 Order the numbers according to their exponents.

The Tertiary period has an exponent of 7. So, it is the youngest period.

Step 2 Order the numbers with the same exponent by comparing the factors.

4.38 > 2.45 > 2.08
Silurian Triassic Jurassic

So, $4.38 \times 10^8 > 2.45 \times 10^8 > 2.08 \times 10^8$.

The time periods ordered from oldest to youngest are Silurian, Triassic, Jurassic, and Tertiary.

Got It? Do this problem to find out.

5. **STEM** Approximately 1.372×10^7 square kilometers of Antarctica and about 1.834×10^6 square kilometers of Greenland are covered by an ice cap. Which land mass has a greater area covered by ice? **Antarctica**

10. 1.4×10^2 h; The number is very large so choosing a larger unit of measure is more meaningful.

Guided Practice

Express each number in standard form. (Example 1)

1. 4.16×10^3 **4160**
2. 3.2×10^{-2} **0.032**
3. 1.075×10^5 **107,500**

Express each number in scientific notation. (Example 2)

4. 1,600,000 **1.6×10^6**
5. 135,000 **1.35×10^5**
6. 0.008 **8×10^{-3}**

Estimate each value using scientific notation. (Example 3) **7–9. Sample answers are given.**

7. 0.000007109 kg **7×10^{-6} kg**
8. 3.7085×10^{14} mL **4×10^{14} mL**
9. 18,900,435 cm **2×10^7 cm**

11. 3.7×10^{-2}, 3.4×10^2, 3.5×10^2, 400

10. If you could walk at the rate of about 1 mile every 20 minutes without stopping, it would take about 1.4×10^2 hours to walk from Columbus, Ohio, to Washington, D.C. Is it more appropriate to report the time as 1.4×10^2 hours or 8.4×10^3 minutes? Explain. (Example 4)

11. Order 3.4×10^2, 3.5×10^2, 3.7×10^{-2}, and 400 from least to greatest. (Example 5)

Example 5

What's the Math? order numbers in scientific notation

- *When ordering numbers written in scientific notation, what must you compare first?* **their exponents**

Need Another Example?

The diameters of Neptune, Saturn, and Uranus are 4.9×10^4 kilometers, 1.2×10^5 kilometers, and 5.1×10^4 kilometers, respectively. Order the planets from greatest to least diameter. **Saturn, Uranus, Neptune**

Formative Assessment

Guided Practice Use these exercises to assess students' understanding of the concept of the lesson. If they need more help, use the Personal Tutors available online.

TICKET Out the Door

Tell students to write how they think what they learned about negative exponents helped them with today's lesson on scientific notation. Ask them to include reasons negative exponents and scientific notation might be useful. They can illustrate with examples. **See students' work.**

3 Practice and Apply

Homework

The **Independent Practice** pages are meant to be used as the homework assignment. If you do not wish to assign the entire exercise set, you can use the table below to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	12–36, 48, 50, 52–76
OL	On Level	13–35 odd, 37–43 odd, 44–48, 50, 52–76
BL	Beyond Level	37–76

Create Your Own Homework Online

SolutionsManual™ can be used to create worksheets for the suggested assignments above, or create your own worksheets for differentiated homework or review.

Watch Out!

Common Error Tell students that when multiplying by 10^x and $x > 0$, move the decimal point to the right. When $x < 0$, move the decimal point to the left. Show students that when the inequality sign points to the right, you move the decimal to the right. When the inequality sign points to the left, you move the decimal to the left.

Independent Practice

Go online for Step-by-Step Solutions eHelp

Express each number in standard form. (Example 1)

12. 6.89×10^4 **68,900**

13. 1.5×10^{-4} **0.00015**

14. 2.3×10^{-5} **0.000023**

15. 9.51×10^{-3} **0.00951**

16. 3.062×10^6 **3,062,000**

17. 7.924×10^2 **792.4**

18. A dollar bill is approximately 1.09×10^{-2} centimeter thick. Write 1.09×10^{-2} in standard form. **0.0109**

19. It is estimated that more than 1.71×10^{11} E-mails are sent each day around the world. Write 1.71×10^{11} in standard form. **171,000,000,000**

Express each number in scientific notation. (Example 2)

20. 700,000 **7×10^5**

21 32,000,000 **3.2×10^7**

22. 0.045 **4.5×10^{-2}**

23. 0.000918 **9.18×10^{-4}**

24. 1,000,000 **1×10^6**

25. 0.006752 **6.752×10^{-3}**

Estimate each value using scientific notation. (Example 3) **26–28. Sample answers are given.**

26. 0.00000095 centimeter **1×10^{-6} cm**

27. 8.375×10^{-23} pound **8×10^{-23} lb**

28. 56,300,001 miles **6×10^7 mi**

29. The distance between Earth and the Moon is about 3.84×10^5 kilometers. Estimate this distance using scientific notation. (Example 3) **Sample answer: 4×10^5 km**

30. The usual growth rate of human hair is 3.3×10^{-4} meter per day. Is it more appropriate to report the rate as 3.3×10^{-4} meter per day or 0.33 millimeter per day? Explain your reasoning. (Example 4)

30. 0.33 millimeter per day; The length is very small, so choosing a smaller unit of measure is more meaningful.

31. One ounce of a certain cheese has 219 milligrams of calcium. Is it more appropriate to include on the nutrition label that the cheese has 2.19×10^{-4} kilogram of calcium or 219 milligrams of calcium? (Example 4) **219 mg**

Order each set of numbers from least to greatest. (Example 5)

32. 2.4×10^2, 2.45×10^{-2}, 2.45×10^2, 2.4×10^{-2} **2.4×10^{-2}, 2.45×10^{-2}, 2.4×10^2, 2.45×10^2**

33. 2.81×10^4, 2805, 2.08×10^5, 3.2×10^4, 3.024×10^2 **3.024×10^2, 2805, 2.81×10^4, 3.2×10^4, 2.08×10^5**

34. 5.9×10^6, 5.9×10^4, 5.01×10^5, 5.1×10^{-3} **5.1×10^{-3}, 5.9×10^4, 5.01×10^5, 5.9×10^6**

35. 9,562,301, 9.05×10^{-6}, 9.5×10^6, 905,000 **9.05×10^{-6}, 905,000, 9.5×10^6, 9,562,301**

36. List the states in the table at the right from least to greatest production of maple syrup. (Example 5) **New Hampshire, Wisconsin, New York, Maine, Vermont**

B **37.** STEM A sheet of gold leaf is approximately 1.25×10^{-5} centimeter thick.

a. Write the value of the thickness as a decimal. **0.0000125 cm**

b. Use the formula $V = \ell wh$ to find the volume in cubic meters of a sheet of gold that is 2 meters wide and 5 meters long. **1.25×10^{-6} m^3**

State	Amount of Syrup Produced (L)
Maine	1.10×10^6
New Hampshire	3.14×10^5
New York	9.65×10^5
Vermont	1.89×10^6
Wisconsin	3.79×10^5

Replace each ● with <, >, or = to make a true statement.

38. 5.72×10^8 ● 5.8×10^8 **<**

39. 35,400 ● 35.4×10^3 **=**

40. 0.042 ● 4.2×10^{-3} **>**

41. 5×10^5 ● 5,000,000 **<**

42. 27,000 ● 2.76×10^3 **>**

43. 6.4×10^{-5} ● 0.000649 **<**

44. CCSS **Justify Conclusions** A news article reported the population of Illinois to be 12,869,257 and the estimated population of Indiana to be 6×10^6.

a. Estimate the number of people living in Illinois. Express the number in scientific notation. **Sample answer: about 1×10^7 people**

b. Which state, Illinois or Indiana, has the greater population? Explain your reasoning. **See margin.**

c. Estimate the combined population of the two states. Express the number in scientific notation. **1.6×10^7**

45. STEM The Moon travels around the Earth at a speed of about 3.68×10^3 kilometers per hour. If the Moon orbits the Earth every 27.3 days, about how far does it travel in one orbit around the Earth? **Sample answer: about 2.41×10^6 km**

46. The speed of light is about 3×10^5 kilometers per second. The distance between Earth and the Moon is about 3.84×10^5 kilometers. Find how long it would take for light to travel from Earth to the Moon. **1.28 s**

47 In a recent year, U.S. route 59 in the Houston area averaged approximately 338,510 vehicles per day. About how many vehicles was this during the entire year? Write the number in scientific notation. Verify your solution by using estimation. **Sample answer: about 1.24×10^8**

H.O.T. Problems Higher Order Thinking

C **48.** CCSS **Identify Structure** Write two numbers in scientific notation with different exponents. Then find the sum, difference, product, and quotient of the two numbers. Write the answers in scientific notation. **See margin.**

49. CCSS **Persevere with Problems** A *googol* is a number that is 1 followed by 100 zeros. A *centillion* is a number that is 1 followed by 303 zeros. Write each of these numbers in scientific notation. **googol: 1×10^{100}; centillion: 1×10^{303}**

50. CCSS **Justify Conclusions** Miami is the second most populous city in Florida. **See margin.**

a. Which number better describes the population of Miami: 3.8×10^4 or 3.8×10^6? Explain.

b. Express Miami's population in another form.

c. Which notation is best to use when describing population? Explain.

51. CCSS **Persevere with Problems** Which number is twice as great as 3×10^2: 6×10^2, 3×10^4, or 6×10^4? Explain. **6×10^2; $3 \times 10^2 = 300$ and $2 \times 300 = 600$ or 6×10^2**

52. **Building on the Essential Question** Your friend thinks 7.8×10^3 is greater than 6.5×10^2 because $7.8 > 6.5$. Explain why your friend's reasoning is incorrect. **See margin.**

CCSS MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	**49, 51**
3 Construct viable arguments and critique the reasoning of others.	**44, 50**
7 Look for and make use of structure.	**48**

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and to apply mathematics to real-world situations.

Additional Answers

44b. Illinois; Sample answer: Both numbers have a single digit times an integer power of 10. The power of 10 for the population of Illinois is greater than the power of 10 for the population of Indiana, so $1 \times 10^7 > 6 \times 10^6$.

48. Sample answer: 2×10^2 and 4×10^3; sum: 4.2×10^3; difference: -3.8×10^3; product: 8×10^5; quotient: 5×10^{-2}

50a. 3.8×10^6; 3.8×10^4 is only about 40,000 people, which is not very many for the second-largest city in Florida.

50b. Sample answer: 3,800,000 or 3.8 million

50c. Sample answer: 3.8 million is easier to read and understand than the standard form (3,800,000) or scientific notation (3.8×10^6).

52. Sample answer: 7.8×10^3 is greater than 6.5×10^2 because the exponent of 3 is greater than the exponent of 2.

Standardized Test Practice

Diagnose Student Errors

Survey student responses for each item. Class trends may indicate common errors and misconceptions.

53.	**A**	counted the leading 0 when finding the exponent to use
	B	correct
	C	used the wrong sign for the exponent
	D	does not understand how to find the power of 10 when writing a number in scientific notation
54.	**F**	incorrectly moved the decimal point
	G	correct
	H	incorrectly moved the decimal point
	J	incorrectly moved the decimal point

Standardized Test Practice

53. The slowest land mammal is the three-toed sloth, which moves 0.07 mile per hour. Which expression represents this number? **B**

A 7×10^{-3} **C** 7×10^{2}

B 7×10^{-2} **D** 7×10^{3}

54. The distance from Earth to the sun is about 9.5×10^{7} miles. Which of the following represents this distance? **G**

F 9,500,000 mi

G 95,000,000 mi

H 950,000,000 mi

J 9,500,000,000 mi

55. **Short Response** The weight of a fruit fly is about 1.3×10^{-4} pound. Estimate the weight of a fruit fly. Express the weight in the form of a single digit times an integer power of 10. **1×10^{-4} lb**

56. **Short Response** A 45-acre farm produces 366,400 pounds of avocados per year. Write an expression, in scientific notation, for the number of pounds of avocados produced per year. **3.664×10^{5} lb**

Common Core Review

57. **STEM** Which type of molecule in the table has a greater mass? How many times greater is it than the other type? 8.EE.1 **penicillin; 10^{5} times greater**

Molecule	Mass (kg)
penicillin	10^{-18}
insulin	10^{-23}

Find each product or quotient. Express using exponents. 8.EE.1

58. $a \cdot a^{5}$ **a^{6}**

59. $(n^{4})(n^{4})$ **n^{8}**

60. $-3x^{2}(4x^{3})$ **$-12x^{5}$**

61. $\frac{3^{8}}{3^{-5}}$ **3^{13}**

62. Use the table to write an expression that shows how many bushels of green beans were picked. Simplify the expression. 7.EE.1 **$78 + g$**

Bushels of Green Beans Picked	
Acre A	40
Acre B	38
Acre C	g

63. Translate *eight times the product of six times a number, all divided by 2* into an algebraic expression. Then simplify the expression. 6.EE.2a **$[8(6n)] \div 2$; $24n$**

64. **Financial Literacy** Of Quincy's monthly paycheck, $\frac{7}{25}$ goes to pay his mortgage, and $\frac{9}{20}$ goes to pay his college loans. Does a greater fraction of his paycheck go to paying his mortgage or his college loans? 8.NS.1 **college loans**

Write each expression using a positive exponent. 8.EE.1

65. 3^{-2} **$\frac{1}{3^{2}}$**

66. h^{-8} **$\frac{1}{h^{8}}$**

67. $(-5)^{-1}$ **$\frac{1}{(-5)^{1}}$**

68. x^{-12} **$\frac{1}{x^{12}}$**

69. 5^{-3} **$\frac{1}{5^{3}}$**

70. $(-7)^{-4}$ **$\frac{1}{(-7)^{4}}$**

Write each expression using exponents. 8.EE.1

71. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **4^{5}**

72. $(6 \cdot 6 \cdot 6) \cdot 6$ **6^{4}**

73. $3 \cdot 2 \cdot 3 \cdot 2 \cdot 2$ **$2^{3} \cdot 3^{2}$**

74. $7 \cdot 7 \cdot 8 \cdot 8 \cdot 8 \cdot 7$ **$7^{3} \cdot 8^{3}$**

75. $10 \cdot 5 \cdot 10 \cdot 5 \cdot 5 \cdot 5$ **$10^{2} \cdot 5^{4}$**

76. $(2 \cdot 2) \cdot 2 \cdot 2 \cdot 2$ **2^{5}**

ISG Interactive Study Guide

See page 81 for:
• Mid-Chapter Check