

2 Teach the Concept

Objectives add, subtract, multiply, and divide numbers written in scientific notation

Building on the Essential Question

At the end of the lesson, students should be able to answer “How does writing numbers in different ways help to make it easier to compute with very large or very small numbers?”

Example 1

What’s the Math? multiplication with scientific notation

- *What do you need to find?* the total number of ants in the Amazon rain forest
- *To find the total number of ants in the Amazon rain forest, what do you need to do?* Multiply the number of ants in one acre of the Amazon rain forest by the total number of acres in the Amazon rain forest.
- *What properties allow you to move and regroup the factors in the expression?* Commutative and Associative Properties
- *When simplifying $10^6 \times 10^9$, do you add, subtract, multiply, or divide the exponents?* add

Need Another Example?

A dime is 1.35×10^{-3} meter thick. What would the height of a stack of one million dimes be in scientific notation? 1.35×10^3

Lesson 4-5

Compute with Scientific Notation

Interactive Study Guide

See page 83–84 for:

- Getting Started
- Vocabulary Start-up
- Real-World Link

Essential Question

Why is it useful to write numbers in different ways?

Common Core State Standards

Content Standards
8.EE.1, 8.EE.3, 8.EE.4

Mathematical Practices
1, 3, 4, 5, 7

What You’ll Learn

- Multiply and divide numbers in scientific notation.
- Add and subtract numbers in scientific notation.

Real-World Link

Aircraft The SR-71 Blackbird is one of the world’s fastest airplanes. It is capable of traveling at a cruising speed of Mach 3, or three times the speed of sound. The speed of sound is approximately 760 miles per hour.



Multiplication and Division with Scientific Notation

You can apply the Product of Powers and Quotient of Powers properties to multiply and divide numbers written in scientific notation.



Example 1



STEM Scientists estimate that there are over 3.5×10^6 ants per acre in the Amazon rain forest. If the Amazon rain forest covers approximately 1 billion acres, find the total number of ants. Write in scientific notation.

Step 1 Write the number of acres in scientific notation.
 $1 \text{ billion} = 1 \times 10^9$

Step 2 Multiply the number of ants per acre by the number of acres to find the total number of ants.

$$\begin{aligned}(3.5 \times 10^6) \times (1 \times 10^9) &= (3.5 \times 1) \times (10^6 \times 10^9) && \text{Commutative and Associative Properties} \\ &= (3.5) \times (10^6 \times 10^9) && \text{Multiply 3.5 by 1.} \\ &= 3.5 \times 10^{6+9} && \text{Product of Powers} \\ &= 3.5 \times 10^{15} && \text{Add the exponents.}\end{aligned}$$

So, there are about 3.5×10^{15} ants in the Amazon rain forest.

Got It? Do these problems to find out.

1a. 3.7653×10^{15} **1b.** 2.20629×10^{-10}
Evaluate each expression. Express the result in scientific notation.

1a. $(4.62 \times 10^5)(8.15 \times 10^9)$

1b. $(7.53 \times 10^{-8})(2.93 \times 10^{-3})$

1c. $(1.2 \times 10^7)(1500)$ 1.8×10^{10}

1d. $(6.4 \times 10^{-5})(12,000)$ 7.68×10^{-1}

Example 2

Evaluate $\frac{7.56 \times 10^8}{3.15 \times 10^3}$. Express the result in scientific notation.

$$\begin{aligned} \frac{7.56 \times 10^8}{3.15 \times 10^3} &= \left(\frac{7.56}{3.15}\right) \left(\frac{10^8}{10^3}\right) && \text{Associative Property} \\ &= 2.4 \left(\frac{10^8}{10^3}\right) && \text{Divide 7.56 by 3.15.} \\ &= 2.4 \times 10^{8-3} && \text{Quotient of Powers} \\ &= 2.4 \times 10^5 && \text{Subtract the exponents.} \end{aligned}$$

Got It? Do these problems to find out.

Evaluate each expression. Express the result in scientific notation.

2a. $\frac{4.62 \times 10^5}{1.4 \times 10^{-9}}$ **3.3×10^{14}** 2b. $\frac{2.5627 \times 10^{-9}}{5.23 \times 10^{-3}}$ **4.9×10^{-7}**

**Example 3**

In 2010, the population of China was about 1.3×10^9 . According to census data, the population of the United States was 308,745,538. About how many times greater was the population of China than the population of the United States in 2010?

Estimate the population of the United States and write in scientific notation.

$$308,745,538 \approx 300,000,000 \text{ or } 3 \times 10^8$$

Find $\frac{1.3 \times 10^9}{3 \times 10^8}$.

$$\begin{aligned} \frac{1.3 \times 10^9}{3 \times 10^8} &= \left(\frac{1.3}{3}\right) \left(\frac{10^9}{10^8}\right) && \text{Associative Property} \\ &\approx 0.4 \times \left(\frac{10^9}{10^8}\right) && \text{Divide 1.3 by 3. Round to the nearest tenth.} \\ &\approx 0.4 \times 10^{9-8} && \text{Quotient of Powers} \\ &\approx 0.4 \times 10^1 && \text{Subtract the exponents.} \\ &\approx 4 \times 10^0 && \text{Write in scientific notation.} \end{aligned}$$

So, the population of China was about 4 times greater than the population of the United States in 2010.

Got It? Do this problem to find out.

3. Until 2008, the world's largest working cattle ranch was located in Australia. It was about 6×10^6 acres. The largest ranch in the United States is 825,000 acres. About how many times larger was the ranch in Australia than the largest ranch in the United States? **about 7 times larger**

Decimal Point

Since 0.4×10^1 is not written in scientific notation, move the decimal point 1 place to the right and subtract one from the exponent.

Example 2

What's the Math? division with scientific notation

- *What property of powers can you apply to evaluate this expression?* **Quotient of Powers Property**
- *When simplifying $\frac{10^8}{10^3}$, do you add, subtract, multiply, or divide the exponents?* **subtract**

Need Another Example?

Evaluate $\frac{8.32 \times 10^7}{1.3 \times 10^5}$. Express the result in scientific notation. **6.4×10^2**

Example 3

What's the Math? solve problems with scientific notation

- *What do you need to find?* **about how many times greater China's population was in 2010 than the United States' population**
- *To find how many times greater China's population was than the United States' population in 2010, what do you need to do?* **Divide China's population in 2010 by the United States' population in 2010.**
- *What is 308,745,538 rounded to the nearest hundred million?* **300,000,000**
- *Why isn't the expression 0.4×10^1 considered to be written in scientific notation?* **the factor is not greater than or equal to 1 and less than 10**

Need Another Example?

The largest planet in our solar system is Jupiter with a diameter of about 143,000 kilometers. The smallest planet in our solar system is Mercury with a diameter of about 5×10^3 kilometers. About how many times greater is the diameter of Jupiter than the diameter of Mercury? **Sample answer: 3×10^1 or 30 times greater**

Example 4

What's the Math? addition and subtraction with scientific notation

- In Example 4a, why do you need to rewrite 3.12×10^4 as 31.2×10^3 ? When adding and subtracting with scientific notation, you need to be able to line up the place values. In order to do that, the numbers must be multiplied by the same power of 10.
- What do you need to do first to add the numbers in Example 4b? Write 46,500 in scientific notation.
- In Example 4b, why is -23.15×10^4 not the correct answer? It is not written in scientific notation.
- How do you rewrite -23.15×10^4 in scientific notation? Move the decimal one place left and add one to the exponent.

Need Another Example?

Evaluate each expression. Express the result in scientific notation.

- 4a. $(7.3 \times 10^5) + (2.4 \times 10^6)$ 3.13×10^6
 4b. $(6,450,000,000) - (8.27 \times 10^7)$ 6.3673×10^9

Formative Assessment

Guided Practice Use these exercises to assess students' understanding of the concept of the lesson. If they need more help, use the Personal Tutors available online.

TICKET Out the Door

Have students find the product of 6×10^5 and 2.5×10^7 . 1.5×10^{13}

Addition and Subtraction with Scientific Notation

When adding or subtracting decimals in standard form, you line up the place values. When adding or subtracting in scientific notation, the place value is represented by the exponent. Each exponent must have the same value in order to add or subtract.

Example 4

Evaluate each expression. Express the result in scientific notation.

a. $(5.45 \times 10^3) + (3.12 \times 10^4)$

$$\begin{aligned} &(5.45 \times 10^3) + (3.12 \times 10^4) \\ &= (5.45 \times 10^3) + (31.2 \times 10^3) \\ &= (5.45 + 31.2) \times 10^3 \\ &= 36.65 \times 10^3 \\ &= 3.665 \times 10^4 \end{aligned}$$

Write 3.12×10^4 as 31.2×10^3 .

Distributive Property

Add 5.45 and 31.2.

Write 36.65×10^3 in scientific notation.

b. $(2.78 \times 10^5) - (46,500)$

$$\begin{aligned} &(2.78 \times 10^5) - (46,500) \\ &= (2.78 \times 10^5) - (4.65 \times 10^4) \\ &= (27.8 \times 10^4) - (4.65 \times 10^4) \\ &= (27.8 - 4.65) \times 10^4 \\ &= 23.15 \times 10^4 \\ &= 2.315 \times 10^5 \end{aligned}$$

Write 46,500 in scientific notation.

Write 2.78×10^5 as 27.8×10^4 .

Distributive Property

Subtract 4.65 from 27.8.

Write 23.15×10^4 in scientific notation.

Got It? Do these problems to find out.

4a. $(1.7 \times 10^7) + (6.25 \times 10^5)$

$$1.7625 \times 10^7$$

4c. $(2.84 \times 10^{11}) - (5.4 \times 10^9)$

$$2.786 \times 10^{11}$$

4b. $0.00864 + (5.67 \times 10^{-4})$

$$9.207 \times 10^{-3}$$

4d. $0.0000321 - (4.9 \times 10^{-7})$

$$3.161 \times 10^{-5}$$

Guided Practice

1. About 1×10^6 fruit flies weigh 1.3×10^2 pounds. How much does one fruit fly weigh? Write in scientific notation. (Example 1) **about 1.3×10^{-4} lb**

Evaluate each expression. Express the result in scientific notation. (Examples 2 and 4)

2. $(1.217 \times 10^5) - (5.25 \times 10^4)$
 6.92×10^4

3. $(2.003 \times 10^4) + (7.98 \times 10^7)$
 7.982003×10^7

4. $\frac{8.25 \times 10^{10}}{2.75 \times 10^4}$ 3×10^6

5. $(3.45 \times 10^7) - (24,650,000)$
 9.85×10^6

6. $523 + (6.2 \times 10^3)$
 6.723×10^3

7. $\frac{9.02 \times 10^3}{4.1 \times 10^5}$ 2.2×10^{-2}

8. The equatorial circumference of Earth is about 4×10^4 kilometers. The equatorial circumference of Jupiter is about 439,263.8 kilometers. About how many times greater is Jupiter's circumference than Earth's? (Example 3) **about 10 times greater**

3 Practice and Apply

Independent Practice

Go online for Step-by-Step Solutions



9. The United States has the most miles of roads in the world at about 4×10^6 miles. Japan has about 7.3×10^5 miles. How many more miles of roads does the United States have than Japan? Write in scientific notation. (Example 1) **about 3.27×10^6 mi**
10. The speed of light is about 1.9×10^5 miles per second. It takes about 500 seconds for light to travel from the sun to Earth. What is the approximate distance between Earth and the sun? Write in scientific notation. (Example 1) **about 9.5×10^7 mi**

Evaluate each expression. Express the result in scientific notation. (Examples 1, 2, and 4)

11. $(5.32 \times 10^8)(3.54 \times 10^3)$ **1.88328×10^{12}**
12. $(1.48 \times 10^{-5})(6.5 \times 10^{-6})$ **9.62×10^{-11}**
13. $(9.5 \times 10^{-4})(28,400)$ **2.698×10^1**
14. $(0.042)(3.15 \times 10^4)$ **1.323×10^3**
15. $\frac{4.97 \times 10^6}{7.1 \times 10^{-8}}$ **7×10^{13}**
16. $\frac{1.86 \times 10^8}{3.1 \times 10^{-4}}$ **6×10^{11}**
17. $\frac{4.7 \times 10^9}{376}$ **1.25×10^7**
18. $\frac{99,500}{5 \times 10^2}$ **1.99×10^2**
19. $(3.205 \times 10^3) + (5.83 \times 10^5)$ **5.86205×10^5**
20. $6,263,000 + (5.4 \times 10^8)$ **5.46263×10^8**
21. $(2.764 \times 10^8) - (6.2 \times 10^7)$ **2.144×10^8**
22. $(9.518 \times 10^7) - 22,000$ **9.5158×10^7**
23. $(4.21 \times 10^{-3})(56,200)$ **2.36602×10^2**
24. $(8.08 \times 10^6)(3.34 \times 10^3)$ **2.69872×10^{10}**
25. $(7.57 \times 10^2)(1.10 \times 10^5)$ **8.327×10^7**
26. $(0.0159)(5.19 \times 10^{-3})$ **8.2521×10^{-5}**
27. The diameter of Mars is about 7×10^6 meters. A standard table tennis ball is 0.04 meter in diameter. About how many times greater is the diameter of Mars than that of a table tennis ball? (Example 3) **about 1.75×10^8 times greater**
28. The United States has a total area (including water) of about 9,826,630 square kilometers. Rhode Island is the smallest state with an area (including water) of about 4×10^3 square kilometers. About how many times greater is the area of the United States than the area of Rhode Island? (Example 3) **about 2500 times greater**
29. Earth is 1.55×10^8 kilometers from the sun. Mercury is 5.80×10^7 kilometers from the sun. Find the difference in distances and express your answer in scientific notation. **9.7×10^7 km**
30. **STEM** Each minute, there are approximately 6×10^3 flashes of lightning around the world. The air around a lightning bolt is heated to about 5.4×10^4 degrees Fahrenheit, which is about five times hotter than the sun. Write each answer in scientific notation and in standard form.
- About how many flashes of lightning are there in a day? **8.64×10^6 ; 8,640,000**
 - About how hot is the sun in degrees Fahrenheit? **1.08×10^4 ; 10,800**
31. A music Web site recently announced that over 4×10^9 songs have been downloaded. It also announced that it has 5×10^7 registered users. Find the average number of downloads per user and express your answer in scientific notation. **8×10^1 downloads**

Homework

The **Independent Practice** pages are meant to be used as the homework assignment. If you do not wish to assign the entire exercise set, you can use the table below to select appropriate exercises for your students' needs.

Differentiated Homework Options

AL	Approaching Level	9–28, 43, 44, 46–68
OL	On Level	9–27 odd, 29–34, 35–41 odd, 43, 44, 46–68
BL	Beyond Level	29–68



Create Your Own Homework Online

SolutionsManual can be used to create worksheets for the suggested assignments above, or create your own worksheets for differentiated homework or review.



MATHEMATICAL PRACTICES

Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	45
3 Construct viable arguments and critique the reasoning of others.	44
5 Use appropriate tools.	32
7 Look for and make use of structure	43

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and to apply mathematics to real-world situations.

Additional Answer

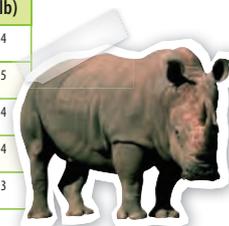
46. Sample answer: When you compute with very large or very small numbers written in standard notation, it can be difficult to keep track of the place value. Scientific notation is a concise way of expressing very large or very small numbers, making it easier to keep track of place value when computing.

Watch Out!

Common Error For Exercise 40, remind students to make sure that their answer is in scientific notation. After writing 180,000 in scientific notation, students may divide $\frac{1.8 \times 10^5}{5 \times 10^8}$ and get the quotient 0.36×10^{-3} . If this happens, tell students that the quotient needs to be written in scientific notation, which means that it must be in the form $a \times 10^n$, where $1 \leq a < 10$.

32. Use Math Tools The table shows the weights of various marine and land animals.

Mammal	Weight (lb)
African elephant	1.44×10^4
blue whale	2.87×10^5
fin whale	9.92×10^4
right whale	8.82×10^4
white rhinoceros	7.94×10^3



- Which animal is about 10 times lighter than a right whale? **white rhinoceros**
- About how many times heavier is the blue whale than the African elephant? **20 times**
- Estimate the combined weight of the fin whale, right whale, and white rhinoceros. Write the combined weight in scientific notation and in standard form. **2.0×10^5 ; 200,000**

33. The average width of a human hair is 4×10^{-3} centimeter. If the cross section of the average hair is round, use the formula $A = 3.14r^2$ to find the approximate area of the cross section of a hair. Write your answer in scientific notation. **$1.256 \times 10^{-5} \text{ cm}^2$**

34. A contractor is using a blend of two different types of sand for a new sand volleyball court. He is using 1.6×10^3 cubic feet of sand that weighs 95 pounds per cubic foot and 1.25×10^3 cubic feet of sand that weighs 88 pounds per cubic foot. How many tons of sand is being used for the volleyball court? **131 tons**

Evaluate each expression. Express the result in scientific notation.

35. $\frac{(2.8 \times 10^{-7})(14,000,000,000)}{3.92 \times 10^4}$ **1×10^{-1}**

36. $\frac{(9.6 \times 10^{20})(3 \times 10^6)}{2 \times 10^5}$ **1.44×10^{22}**

37. $\frac{1.86 \times 10^8}{3.1 \times 10^{-4}} + 5.4 \times 10^{10}$ **6.54×10^{11}**

38. $\frac{4.5 \times 10^4}{75,000,000} \times (4.9 \times 10^6)$ **2.94×10^2**

39. $\left(\frac{6 \times 10^{-100}}{2.5 \times 10^{-60}}\right)(3.7 \times 10^{15})$ **8.88×10^{-25}**

40. $\left(\frac{180,000}{5 \times 10^8}\right)(9 \times 10^2)$ **3.24×10^{-1}**

41. $(8.2 \times 10^4 + 8,249) \times 10^8$ **9.0249×10^{12}**

42. $(5.29 \times 10^4 - 52,000) \times 10^5$ **9×10^7**



H.O.T. Problems Higher Order Thinking

43. Identify Structure Write an addition expression and a subtraction expression, each with a value of 2.4×10^{-3} . **Sample answers: $(2.15 \times 10^{-3}) + (2.5 \times 10^{-4})$; $(2.56 \times 10^{-3}) - (1.6 \times 10^{-4})$**

44. Which One Doesn't Belong? Identify the expression that does not belong with the other three. Explain your reasoning. **52.5×10^8 does not belong. $52.5 \times 10^8 = 5.25 \times 10^9$, while the three remaining expressions each equal 5.25×10^7 .**

5.25×10^7

$(2.1 \times 10^2)(2.5 \times 10^5)$

52.5×10^8

$(2.1)(2.5) \times 10^{(2+5)}$

45. Persevere with Problems There are about 2.5×10^{10} red blood cells in the average adult. A googol is 1×10^{100} . About how many adults would it take to have a total of 1 googol red blood cells? **about 4×10^{89} adults**

46. Building on the Essential Question How does writing numbers in different ways help to make it easier to compute with very large or very small numbers? **See margin.**

**Standardized Test Practice**

47. Ariana is evaluating $(8 \times 10^3) + (4 \times 10^5)$, as shown below.

$$\begin{aligned} &(8 \times 10^3) + (4 \times 10^5) \\ &(8 + 4) + (10^3 + 10^5) \\ &12 + (10^3 + 10^5) \\ &12 + 10^8 \end{aligned}$$

What should Ariana have done differently to evaluate the expression correctly? **A**

- A** made both numbers have the same power of 10
B subtracted the exponents
C multiplied 8×4 instead of adding $8 + 4$
D made the last line 12×10^8
48. What is the value of $(2.8 \times 10^3)(1,600,000)$? **J**
- F** 4.48×10^{18} **H** 44.8×10^9
G 4.48×10^6 **J** 4.48×10^9

49. After its first year in business, a movie Web site announced that over 500,000,000 movies were downloaded by 4×10^6 registered users. What is the average number of movies per user? **B**

- A** about 1.25×10^{-2} movies
B about 125 movies
C 1.25×10^3 movies
D about 12.5 movies

50. **Short Response** Earth is 1.55×10^8 kilometers from the sun. Venus is 109 million kilometers from the sun. Find the difference in distances and express your answer in scientific notation.
 4.6×10^7 km

**Common Core Review**

Write an integer for each situation. Then identify its opposite. 7.NS.1a

51. 58°C below zero **-58 ; $+58$ or 58**
52. 15 gallons per mile more than usual **$+15$ or 15 ; -15**
53. a withdrawal of \$4500 **$-4500$; $+4500$ or 4500**
54. a scuba diver's descent of 50 feet **-50 ; $+50$ or 50**
55. a bank deposit of \$68.00 **$+68$ or 68 ; -68**
56. an airplane's ascent of 300 feet **$+300$ or 300 ; -300**

Complete each expression. 7.NS.1c

57. $18 - 5 = 18 + \blacksquare$ **-5** 58. $\blacksquare - (-3) = 12$ **9** 59. $12 = 10 - \blacksquare$ **-2**

60. The volume of one cube is 5^3 cubic inches. What is the volume of 3.5 of these cubes? 8.EE.1 **437.5 in^3**
61. The speed of sound is approximately 7.6×10^2 miles per hour. Write 7.6×10^2 in standard form. 8.EE.3 **760**
62. The SR-71 Blackbird is more than 30 years old. It can fly at altitudes above 8×10^4 feet. Is it more appropriate to report the altitude as 8×10^4 feet or as 9.6×10^5 inches? 8.EE.4 **$8 \times 10^4 \text{ ft}$**

Determine whether each equation is true or false. If the equation is false, explain why. 7.NS.2c

63. $3 \times (-4) = -12$ **true**
64. $-\frac{8}{4} = \frac{-8}{-4}$ **false; $-\frac{8}{4} = \frac{-8}{4}$ or $\frac{8}{-4}$**
65. $-15 \div (-3) = 5$ **true**
66. $-36 \div 6 = 6$ **false; $-36 \div (6) = -6$**
67. $-12 \times (-11) = 132$ **true**
68. $-1 \times (-1) = 1$ **true**

**Standardized Test Practice****Diagnose Student Errors**

Survey student responses for each item. Class trends may indicate common errors and misconceptions.

- | | |
|-------|---|
| 47. A | correct |
| B | chose strategy for dividing instead of adding |
| C | did not apply Distributive Property correctly |
| D | replaced incorrect line with another incorrect line |
| 48. F | multiplied exponents instead of adding |
| G | used wrong exponent when writing 1,600,000 in scientific notation |
| H | placed decimal point incorrectly |
| J | correct |
| 49. A | used incorrect sign for exponent |
| B | correct |
| C | added exponents incorrectly |
| D | placed decimal point incorrectly when changing answer from scientific notation to standard form |