2 Teach the Concept

Objectives find and estimate square roots and cube roots

Building on the Essential Question

At the end of the lesson, students should be able to answer "When finding square roots of numbers that are not perfect squares, what is the difference between an exact value and an approximation?"

Example 1

What's the Math? find square roots

- *In Example 1a, are you looking for the positive square root, negative square root, or both?* **positive**
- *What does the negative sign in Example 1b mean?* **to find the negative square root**
- *In Example 1c, what does the plus/minus sign in front of the radical sign mean?* **to find both square roots**
- *In Example 1d, is there an integer that when multiplied by itself is equal to −81?* **no**

Need Another Example?

Find each square root.

1a. $\sqrt{64}$ **8**

1b. $-\sqrt{121}$ **−11**

1c. $\pm\sqrt{256}$ **±16**

1d. $\sqrt{-9}$ **no real solution**

Lesson 4-6

Square Roots and Cube Roots

ISG Interactive Study Guide

See pages 85–86 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

Why is it useful to write numbers in different ways?

CCSS Common Core State Standards

Content Standards
8.NS.2, 8.EE.2

Mathematical Practices
1, 2, 3, 4, 7

Vocabulary

square root
perfect square
radical sign
cube root
perfect cube

What You'll Learn
- Find square roots.
- Find cube roots.

Real-World Link

Rain Forest Tropical rainforests contain the greatest diversity of plants and animals on Earth—and they cover less than 5 percent of Earth's land! Just a four-square-mile patch of rainforest contains thousands of species of plants and trees, and hundreds of species of mammals, birds, reptiles, and amphibians.

Design Pics/Corey Hochachka

Key Concept Square Roots

Words	A **square root** of a number is one of its two equal factors.
Symbols	If $x^2 = y$, then x is a square root of y.

Number like 9, 25, and 144 are **perfect squares**, because they are squares of integers. The opposite of squaring a number is finding the square root.

A **radical sign**, $\sqrt{}$ is used to indicate a nonnegative square root. Every positive number has both a positive and a negative square root.

$$\sqrt{36} = 6 \qquad -\sqrt{36} = -6 \qquad \pm\sqrt{36} = \pm 6 \text{ or } 6, -6$$

A negative number like −36 has no real-number square root because the square of a number cannot be negative. You will learn about real numbers in the next lesson.

Example 1

Tutor

Find each square root.

a. $\sqrt{9}$

$\sqrt{9} = 3$ Find the positive square root of 9; $3^2 = 9$.

b. $-\sqrt{64}$

$-\sqrt{64} = -8$ Find the negative square root of 64; $8^2 = 64$.

c. $\pm\sqrt{4}$

$\pm\sqrt{4} = \pm 2$ Find both square roots of 4; $2^2 = 4$.

d. $\sqrt{-81}$

There is no real square root because no number times itself is equal to −81.

Got It? Do these problems to find out.

1a. $\sqrt{49}$ **7**

1b. $-\sqrt{16}$ **−4**

1c. $\pm\sqrt{100}$ **±10**

1d. $\sqrt{-49}$ **no real solution**

You can estimate the square root of an integer that is not a perfect square by determining between which two consecutive integers the square root lies.

Example 2

Tools Tutor

Estimate each square root to the nearest integer.

a. $\sqrt{33}$

The largest perfect square less than 33 is 25. $\sqrt{25} = 5$

The smallest perfect square greater than 33 is 36. $\sqrt{36} = 6$

Plot each square root on a number line. Then estimate $\sqrt{33}$.

$25 < 33 < 36$ — Write an inequality.

$5^2 < 33 < 6^2$ — $25 = 5^2$ and $36 = 6^2$

$\sqrt{5^2} < 33 < \sqrt{6^2}$ — Find the square root of each number.

$5 < \sqrt{33} < 6$ — Simplify.

So, $\sqrt{33}$ is between 5 and 6. Since 33 is closer to 36 than to 25, the best integer estimate for $\sqrt{33}$ is 6.

Check Check using a calculator.

2nd [√] 33 ENTER **5.744562647**

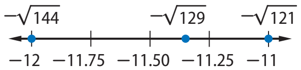
$\sqrt{33} \approx 6$ ✓

b. $-\sqrt{129}$

The largest perfect square less than 129 is 121. $\sqrt{121} = -11$

The smallest perfect square greater than 129 is 144. $\sqrt{144} = -12$

The negative square root of 129 is between the integers −11 and −12. Plot each square root on a number line. Then estimate $-\sqrt{129}$.



So, $-\sqrt{129}$ is between −12 and −11. Since 129 is closer to 121 than to 144, the best integer estimate for $-\sqrt{129}$ is −11.

Check Check using a calculator.

(−) 2nd [√] 129 ENTER **−11.357816691**

$\sqrt{-129} \approx -11$ ✓

Watch Out! When keying your calculator to find the negative of a square root, for example $-\sqrt{129}$, be sure to use the (−) button instead of the − button for the negative sign.

Got It? Do these problems to find out.

2a. $\sqrt{60}$ **8**

2b. $-\sqrt{23}$ **−5**

2c. $\sqrt{14}$ **4**

2d. $-\sqrt{79}$ **−9**

Example 2

What's the Math? estimate square roots

- *In Example 2a, what is the largest perfect square that is less than 33 and the smallest perfect square that is greater than 33?* **25 and 36**
- *What are $\sqrt{25}$ and $\sqrt{36}$?* **5 and 6**
- *What is the best integer estimate for $\sqrt{33}$? Explain.* **6; 33 is closer to 36 than to 25, so $\sqrt{33}$ is closer to $\sqrt{36}$ than to $\sqrt{25}$.**
- *What steps would you need to follow to estimate the square root in Example 2b?* **First, find the largest perfect square less than 129 and the smallest perfect square greater than 129. Then you can use the negative square roots of the perfect squares to estimate $-\sqrt{129}$ to the nearest integer.**

Need Another Example?

Estimate each square root to the nearest integer.

2a. $\sqrt{22}$ **5**

2b. $-\sqrt{319}$ **−18**

Example 3

What's the Math? use square roots to solve problems

- *What do you need to find?* **the distance a person can see when standing on the observation deck of Seattle's Space Needle**
- *To find the distance a person can see when standing on the observation deck of Seattle's Space Needle, what do you need to do?* **Using the formula $d = 1.22 \cdot \sqrt{h}$, substitute 502 for the person's height h from the ground in feet and solve for the distance d to the horizon in miles.**
- *Why do you use the principal square root when finding the square root in this situation?* **You are finding a distance, and a negative answer does not make sense.**

Need Another Example?

The tallest building in Houston, Texas, is the JPMorgan Chase Tower, standing at 1002 feet. About how far to the horizon can a person standing on the top floor see on a clear day? Round your answer to the nearest tenth. **38.6 mi**

When finding square roots in real-world situations, use the positive, or *principal*, square root when a negative answer does not make sense.

Choose a Form

Express a number as a square root if an exact answer is needed. Express a number as a decimal if an approximation is sufficient.

Example 3

Tutor

On a clear day, the number of miles a person can see to the horizon can be found using the formula $d = 1.22 \cdot \sqrt{h}$, where d is the distance to the horizon in miles and h is the person's distance from the ground in feet. The observation deck of Seattle's Space Needle is 520 feet high. How far to the horizon can a person standing on the observation deck see? Round to the nearest tenth.

Estimate The distance is between $1 \cdot \sqrt{400}$ and $1 \cdot \sqrt{900}$. So, it's between 20 and 30.

$d = 1.22 \cdot \sqrt{h}$	Write the equation.
$= 1.22 \cdot \sqrt{520}$	Replace h with 520.
$\approx 1.22 \cdot 22.8$	Use a calculator.
≈ 27.8	Simplify.

The approximate distance to the horizon is 27.8 miles to the nearest tenth.

Check for Reasonableness $20 < 27.8 < 30$ ✓

Got It? Do these problems to find out.

3a. Spring Port Ledge Lighthouse in Maine is approximately 55 feet tall. Calculate about how far a person who is standing at the top of the lightbouse can see on a clear day. Round to the nearest tenth of a mile. **9.0 mi**

3b. The observation deck of the Washington Monument is 500 feet high. Calculate about how far a person on the observation deck can see on a clear day. Round to the nearest tenth of a mile. **27.3 mi**

Key Concept Cube Roots

Words	A cube root of a number is one of its three equal factors.
Symbols	If $x^3 = y$, then $x = \sqrt[3]{y}$.
Examples	Since $2 \times 2 \times 2$ or $2^3 = 8$, 2 is a cube root of 8. Since $-6 \times (-6) \times (-6) = -216$, -6 is a cube root of -216.

A **cube root** of a number is one of three equal factors of the number. The symbol $\sqrt[3]{}$ is used to indicate the cube root of a number.

Every integer has exactly one cube root.

- The cube root of a positive number is positive.
- The cube root of zero is zero.
- The cube root of a negative number is negative.

Example 4

Tutor

Find each cube root.

a. $\sqrt[3]{343}$

$\sqrt[3]{343} = 7$ $\quad 7^3 = 7 \cdot 7 \cdot 7$ or 343

b. $\sqrt[3]{-729}$

$\sqrt[3]{-729} = -9$ $\quad (-9)^3 = (-9) \cdot (-9) \cdot (-9)$ or -729

Got It? Do these problems to find out.

4a. $\sqrt[3]{64}$ **4**

4b. $\sqrt[3]{-1331}$ **−11**

You can also estimate cube roots mentally by using **perfect cubes**.

Example 5

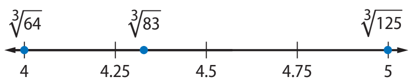
Tutor

Estimate $\sqrt[3]{83}$ to the nearest integer. Do not use a calculator.

$\sqrt[3]{83}$

The first perfect cube less than 83 is 64. $\quad \sqrt[3]{64} = 4$

The first perfect cube greater than 83 is 125. $\quad \sqrt[3]{125} = 5$



The cube root of 83 is between the integers 4 and 5. Since 83 is closer to 64 than to 125, you can expect $\sqrt[3]{83}$ to be closer to 4 than to 5.

Got It? Do these problems to find out.

5a. $\sqrt[3]{72}$ **4**

5b. $\sqrt[3]{-2024}$ **−13**

Guided Practice

Check

Find each square root. (Example 1)

1. $\sqrt{16}$ **4**
2. $-\sqrt{100}$ **−10**
3. $\pm\sqrt{81}$ **±9**

Estimate each square root to the nearest integer. (Example 2)

4. $\sqrt{27}$ **5**
5. $-\sqrt{48}$ **−7**
6. $\pm\sqrt{39}$ **±6**
7. A baseball diamond is actually a square with an area of 8100 square feet. Most baseball teams cover their diamond with a tarp to protect it from the rain. The sides are all the same length. How long is the tarp on each side? (Example 3) **90 ft**

Find each cube root. (Example 4)

8. $\sqrt[3]{512}$ **8**
9. $\sqrt[3]{2197}$ **13**
10. $\sqrt[3]{-1000}$ **−10**
11. $\sqrt[3]{-343}$ **−7**

Estimate each cube root to the nearest integer. (Example 5)

12. $\sqrt[3]{74}$ **4**
13. $\sqrt[3]{39}$ **3**
14. $\sqrt[3]{-636}$ **−9**
15. $\sqrt[3]{-879}$ **−10**

Example 4

What's the Math? find cube roots

- *In Example 4a, what is one of three equal factors with a product of 343?* **7**
- *Why is it possible to find the cube root of a negative number?* **You can use a negative number as a factor three times and the product is negative.**
- *What number when used as a factor three times has a product of −729?* **−9**

Need Another Example?

Find each cube root.

4a. $\sqrt[3]{125}$ **5** $\quad$ **4b.** $\sqrt[3]{-2197}$ **−13**

Example 5

What's the Math? estimate cube roots

- *What is the largest perfect cube that is less than 83?* **64**
- *What is the smallest perfect cube that is greater than 83?* **125**
- *What are $\sqrt[3]{64}$ and $\sqrt[3]{125}$?* **4 and 5**

Need Another Example?

Estimate $\sqrt[3]{47}$ to the nearest integer. **4**

Formative Assessment

Guided Practice Use these exercises to assess students' understanding of the concept of the lesson. If they need more help, use the Personal Tutors available online.

TICKET Out the Door

Have students explain how to estimate the square root of a number to the nearest integer. **See students' work.**

3 Practice and Apply

Homework

The **Independent Practice** pages are meant to be used as the homework assignment. If you do not wish to assign the entire exercise set, you can use the table below to select appropriate exercises for your students' needs.

Differentiated Homework Options		
AL	Approaching Level	16–36, 39, 40, 42–62
OL	On Level	17–35 odd, 37–40, 42–62
BL	Beyond Level	37–62

Create Your Own Homework Online

eSolutionsManual can be used to create worksheets for the suggested assignments above, or create your own worksheets for differentiated homework or review.

CCSS MATHEMATICAL PRACTICES Emphasis On	Exercise(s)
1 Make sense of problems and persevere in solving them.	41
2 Reason abstractly and quantatively.	39
7 Look for and make use of structure.	40

Mathematical Practices 1, 3, and 4 are aspects of mathematical thinking that are emphasized in every lesson. Students are given opportunities to be persistent in their problem solving, to express their reasoning, and to apply mathematics to real-world situations.

Independent Practice

Go online for Step-by-Step Solutions eHelp

Find each square root. (Example 1)

16. $\sqrt{36}$ **6**
17. $\sqrt{9}$ **3**
18. $-\sqrt{169}$ **−13**
19. $-\sqrt{144}$ **−12**
20. $\pm\sqrt{-25}$ **no real solution**
21. $\pm\sqrt{1}$ **±1**

Estimate each square root to the nearest integer. (Example 2)

22. $\sqrt{83}$ **9**
23. $\sqrt{34}$ **6**
24. $-\sqrt{102}$ **−10**
25. $-\sqrt{14}$ **−4**
26. $\pm\sqrt{78}$ **±9**
27. $\pm\sqrt{146}$ **±12**

28. The table shows the heights of the tallest roller coasters at Cedar Point. Use the formula from Example 3 to determine how far a rider can see from the highest point of each ride. Round to the nearest tenth. (Example 3)

a. Millennium Force **21.5 mi**

b. Mean Streak **15.5 mi**

c. How much farther can a rider see on the Top Thrill Dragster than on the Magnum XL-200? **7.5 mi**

Cedar Point Attractions	
Roller Coaster	**Height (ft)**
Mean Streak	161
Magnum XL-200	205
Millennium Force	310
Top Thrill Dragster	420

Find each cube root. (Example 4)

29. $\sqrt[3]{-1728}$ **−12**
30. $\sqrt[3]{-2744}$ **−14**
31. $\sqrt[3]{216}$ **6**
32. $\sqrt[3]{1331}$ **11**

Estimate each cube root to the nearest integer. Do not use a calculator. (Example 5)

33. $\sqrt[3]{499}$ **8**
34. $\sqrt[3]{576}$ **8**
35. $\sqrt[3]{-79}$ **−4**
36. $\sqrt[3]{-1735}$ **−12**

B **37.** The area of a square is 215 square centimeters. Find the length of a side to the nearest tenth. Then find its approximate perimeter. **14.7 cm; 58.8 cm**

38. Order $\sqrt{77}, -8, -\sqrt{83}, 9, -10, -\sqrt{76}, \sqrt{65}$ from least to greatest.
$-10, -\sqrt{83}, -\sqrt{76}, -8, \sqrt{65}, \sqrt{77}, 9$

H.O.T. Problems Higher Order Thinking

C **39.** CCSS **Reason Abstractly** Write a number that completes the analogy.
x^2 is to 121 as x^3 is to __?__. **1331**

40. CCSS **Identify Structure** Find a square root that lies between 17 and 18. **Sample answer: $\sqrt{300}$**

41. CCSS **Persevere with Problems** Use inverse operations to evaluate the following.

a. $(\sqrt{246})^2$ **246**
b. $(\sqrt{811})^2$ **811**
c. $(\sqrt{732})^2$ **732**

42. **Building on the Essential Question** Describe the difference between an exact value and an approximation when finding square roots of numbers that are not perfect squares. Give an example of each. **See Answer Appendix.**