

# Direct Variation



## Interactive Study Guide

See pages 201–202 for:

- Getting Started
- Real-World Link
- Notes



## Essential Question

How are linear functions used to model proportional relationships?



## Common Core State Standards

**Content Standards**  
7.RP.2, 7.RP.2a, 7.RP.2b,  
8.EE.5

**Mathematical Practices**  
1, 3, 4



## Vocabulary

direct variation  
constant of variation

## What You'll Learn

- Identify direct variation.
- Use direct variation to solve problems.



## Real-World Link

**Video Games** According to a recent survey, about 56% of U.S. households own a current video game system. Games for these systems can get expensive, so many stores offer sales on pre-owned video games. The prices for pre-owned games at a local store are shown in the table below. A *direct variation* equation can be used to represent this situation.

Number of Games	Total Cost (\$)
2	16.50
4	33.00
6	49.50
8	66.00



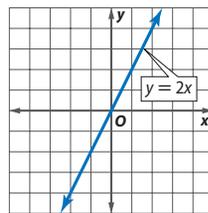
## Key Concept Direct Variation

**Words** A direct variation is a relationship in which the ratio of  $y$  to  $x$  is a constant,  $m$ . We say  $y$  varies directly with  $x$ .

**Symbols**  $\frac{y}{x} = m$  or  $y = mx$ , where  $m \neq 0$

**Example**  $y = 2x$

**Graph**



When the ratio of two variable quantities is constant, their relationship is a **direct variation**. Since  $(0, 0)$  is a solution of  $y = mx$ , the graph of a direct variation always passes through the origin and represents a proportional linear relationship.

In the equation  $y = mx$ ,  $m$  is called the **constant of variation** or constant of proportionality. It is the slope of the graph of  $y = mx$ .



## Example 1



### Directly Proportional

Since  $k$  is a constant rate of change in a direct variation, we can say the following:

- $y$  varies directly with  $x$ .
- $y$  is directly proportional to  $x$ .

The graph shows the cost of different amounts of trail mix. Determine if the relationship between the cost and the weight of the trail mix is a direct variation.

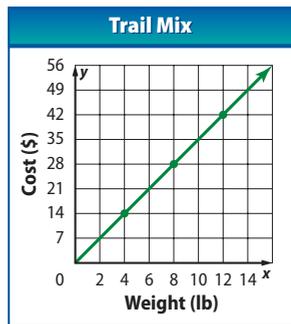
To determine if the relationship is a direct variation, find  $\frac{\text{cost } y}{\text{weight } x}$  for points on the graph.

$$\frac{\$14}{4 \text{ lb}} = \$3.50/\text{lb}$$

$$\frac{\$28}{8 \text{ lb}} = \$3.50/\text{lb}$$

$$\frac{\$42}{12 \text{ lb}} = \$3.50/\text{lb}$$

The ratio,  $\$3.50/\text{lb}$ , is a constant rate. Since the graph passes through the origin and the ratios are constant, the cost of the trail mix varies directly with the weight of the trail mix.



**Got It?** Do this problem to find out.

1. Tyler charges his customers \$10 per week plus \$5 every time he walks their dogs. Determine whether the relationship between total weekly cost and the number of times the dog is walked is a constant variation.

**No; the ratio  $\frac{\text{number of walks}}{\text{pay}}$  is different for every pair of values.**



## Example 2



The equation  $y = 40x$  represents the distance  $y$  in miles an ostrich can travel in  $x$  hours. Determine whether there is a constant of variation. If so, explain what it represents in this situation.

$$y = mx \quad \text{Compare the equation to } y = mx, \text{ where } m \text{ is the constant of variation.}$$

$$y = 40x$$

$$40 = \frac{y}{x}$$

The constant of variation is 40. This means that an ostrich can travel 40 miles per hour.

**Got It?** Do these problems to find out.

- 2a. The equation  $P = 6s$  relates the perimeter  $P$  of a regular hexagon to the length of a side  $s$ . Determine if there is a constant of variation. If so, explain what it represents in this situation. **The constant of variation is 6. The perimeter of a hexagon varies directly with the length of a side.**
- 2b. **STEM** The equation  $K = C + 273$  relates the Kelvin temperature  $K$  to Celsius temperature  $C$ . Determine if there is a constant of variation. If so, explain what it represents in this situation.

**2b. There is no constant of variation. Kelvin temperature does not vary directly with Celsius temperature.**



## Example 3



### Graphs

The graph of a direct variation is always a line that goes through the origin.

**STEM** The time it takes to burn amounts of information on a CD varies directly with the amount of information. Write and solve an equation to find how long it will take to fill a 700-megabyte CD.

**Step 1** Use the equation  $y = mx$ . Choose any point on the graph. Then solve for  $m$ .

$$y = mx \quad \text{Direct variation equation}$$

$$10 = m(2.5) \quad \text{Replace } y \text{ with } 10 \text{ and } x \text{ with } 2.5.$$

$$4 = m \quad \text{Simplify.}$$

**Step 2** Use  $m$  to write an equation.

$$y = mx \quad \text{Direct variation equation}$$

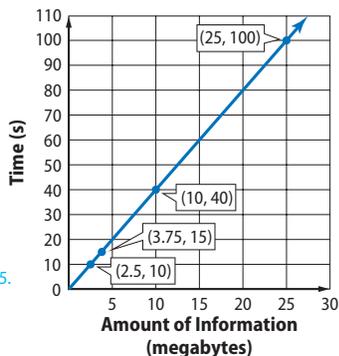
$$y = 4x \quad \text{Replace } m \text{ with } 4.$$

**Step 3** Find how long it will take to fill the CD.

$$y = 4x \quad \text{Write the direct variation equation.}$$

$$y = 4(700) \text{ or } 2800 \quad \text{Replace } x \text{ with } 700 \text{ and simplify.}$$

It will take 2800 seconds to fill the CD.



**Got It?** Do these problems to find out.

3. The cost of bulk peanuts varies directly with the weight of the peanuts. At a local grocery store, 2 pounds of peanuts costs \$5.80. Write and solve an equation to find how much 5 pounds of peanuts would cost.  $y = 2.9x$ ; \$14.50

## Compare Direct Variations



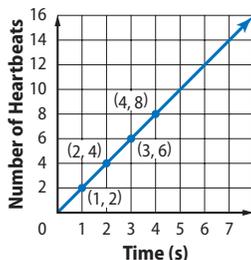
## Example 4



The equation  $y = 1.5x$  represents the relationship between the number of heartbeats  $y$  and time in seconds  $x$  for a dog. The graph shows the heartbeats for a cat. Which animal has a faster heart rate? Explain.

In the equation  $y = 1.5x$ , the slope or unit rate is 1.5 beats per second. In the graph, the point (1, 2) represents the unit rate, which is 2 beats per second. Since  $2 > 1.5$ , the cat has a faster heart rate.

Cat Heart Rate



**4. Rabbit; Sample answer: The unit rate for the rabbit is 3 beats per second, which is greater than 2 beats per second.**

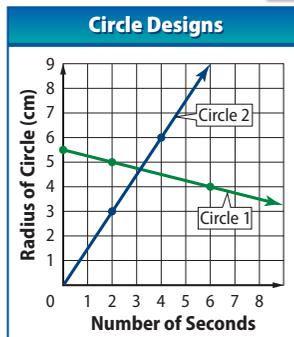
**Got It?** Do this problem to find out.

4. The equation  $y = 3x$  represents the heart rate of a rabbit, where  $x$  is the time in seconds and  $y$  is the number of heartbeats. Does the rabbit or cat have a faster heart rate? Explain.

## Guided Practice



- Recall the graph from Lesson 9–3 of the circular design on an Internet advertisement, shown at the right. The design has two circles, one that is decreasing in size and one that is increasing in size. (Example 1) **a–b. See Answer Appendix.**
  - Determine if the relationship between the number of seconds and the radius of circle 1 is a direct variation.
  - Determine if the relationship between the number of seconds and the radius of circle 2 is a direct variation.
- Financial Literacy** The equation  $y = 750x$  represents the number of dollars  $y$  Olivia earns in  $x$  weeks. Determine if there is a constant of variation. If so, explain what it represents. (Example 2)



- The constant of variation is 750. This means that Olivia earns \$750 per week.**
- STEM** The length that a spring stretches varies directly with the amount of weight attached to it. When an 8-ounce weight is attached, a spring stretches 2 inches. (Example 3)
    - Write an equation relating the weight  $x$  and the amount of stretch  $y$ .  **$y = 0.25x$**
    - Predict the stretch of the spring when it has a 20-ounce weight attached. **5 in.**
  - The table below shows the changes in height for a kitesurfer. Assume that the height varies directly with the number of seconds. The height of a second kitesurfer increases 3.5 feet per second. Does the height of the first or second kitesurfer increase faster? Explain your reasoning. (Example 4)

Time (s)	4	6	8	10
Height (ft)	8	12	16	20

**the second kitesurfer; Sample answer: The unit rate for this kitesurfer is 3.5 feet per second, which is greater than 2 feet per second.**

## Independent Practice

Go online for Step-by-Step Solutions



Determine if the relationship between the two quantities is a direct variation. (Example 1)

- Lifting Weights**

**This is a direct variation since the graph passes through the origin and the ratio of the number of weights to the total weight is constant.**
- Getting Tickets**

**This is not a direct variation since the graph does not pass through the origin and the ratio of tickets available to tickets sold is not constant.**

**7. The constant of variation is 26.2. This means that Conrad runs 26.2 miles in each marathon.**

- The equation  $y = 26.2x$  represents the number of miles  $y$  Conrad runs in  $x$  marathons. Determine if there is a constant of variation. If so, explain what it represents. (Example 2)

8. The equation  $y = 3.50x + 5$  represents the number of dollars  $y$  Kristin charges for driving you  $x$  miles in her taxi. Determine if there is a constant of variation. If so, explain what it represents. (Example 2) **There is no constant of variation. The cost to ride in Kristin's taxi does not vary directly with the number of miles she takes you.**

**9 STEM** Water pressure is measured in pounds per square inch (psi), which varies directly with the depth of the water. (Example 3)

- a. Write an equation that relates the depth and the water pressure. Round to the nearest tenth.  **$y = 0.4x$**
- b. The deepest dive ever recorded by an orca whale is 900 feet. What is the approximate water pressure at this depth? **360 psi**

Depth (ft)	Pressure (psi)
$x$	$y$
33	14.7
66	29.4
99	44.1
132	58.8

10. **CCSS Model with Mathematics** The cost of cheese varies directly with the number of pounds bought. Suppose 2 pounds cost \$8.40. Write and solve an equation to find the cost of 3.5 pounds of cheese. (Example 3)  **$y = 4.2x$ ; \$14.70**

11. Erica's earnings  $y$  varies directly with the number of hours she works  $x$ . The relationship is shown in the table below. Javier's earning can be represented by the equation  $y = 9.2x$ . Who earns more money per hour? Explain. (Example 4)

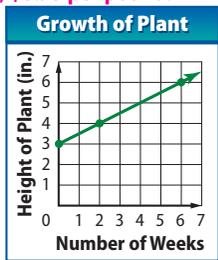
Hours	4	5	6	7
Money Earned (\$)	40.60	50.75	60.90	71.05

**Erica; Sample answer: The unit rate for her pay is \$10.15 per hour, which is more than \$9.20 per hour.**

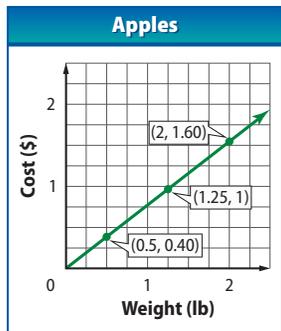
12. The graph shows the cost  $y$  of  $x$  pounds of apples. The cost of  $x$  pounds of pears can be represented by the equation  $y = 0.95x$ . Which fruit has the lower price? Explain your reasoning. (Example 4)

**apples; Sample answer: The unit cost for apples is \$0.80 per pound, which is less than the unit cost for pears, \$0.95 per pound.**

- B** 13. Use the graph at the right to determine whether each statement is *true* or *false*. Explain your reasoning.



- a. There is a direct variation.
- b. There is a linear relationship.
- true; Sample answer: the rate of change is constant.**



**13a. false; Sample answer: the graph does not pass through the origin.**

15. **Sample answer: Compare the equation to  $y = mx$ , where  $m$  is the constant of variation.  $y = 20x$ , where 20 is the constant of variation.  $y = 20x + 1$  does not have a constant of variation.**

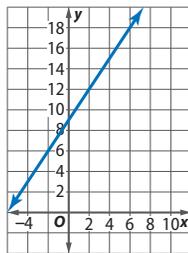
**H.O.T. Problems** Higher Order Thinking

- C** 14. **CCSS Find the Error** Ramiro is determining whether a line through the points with coordinates (2, 12) and (6, 18) represents a direct variation relationship. Find his mistake and correct it.

$$\frac{\text{change in } y}{\text{change in } x} = \frac{18 - 12}{6 - 2}$$

$$= \frac{6}{4} \text{ or } 1.5$$

The relationship is a direct variation.



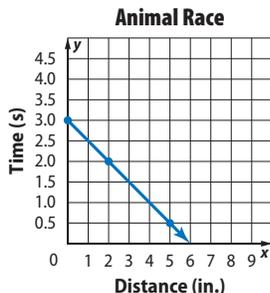
**14. Ramiro incorrectly said the relationship was a direct variation. It is not. Graphs that are direct variation must go through the origin.**

15. **Building on the Essential Question** Describe the steps you take to determine whether an equation has a constant of variation. Give an example of an equation that has a constant of variation and one that does not.



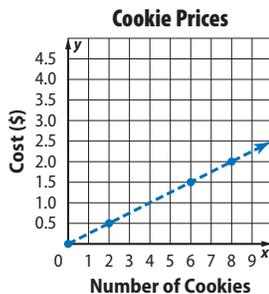
## Standardized Test Practice

16. Which is *not* a true statement about the graph shown below? **B**



- A It is a linear relationship.  
 B The constant of variation is 1.  
 C The slope is  $-\frac{1}{2}$ .  
 D The x-intercept is (6, 0).
17. The equation  $y = 32x$  describes the number of miles per gallon that Melanie's car gets. Predict how many miles Melanie can drive on 12 gallons of gas. **H**
- F 44 mi                      H 384 mi  
 G 352 mi                     J 416 mi

18. The cost of cookies at a bake sale is shown below. **B**



- Which of the following is the best prediction for the cost of 21 cookies?
- A \$5.00                      C \$5.50  
 B \$5.25                      D \$5.75
19. **Short Response** At an amusement park, the cost of admission varies directly with the number of tickets purchased. One ticket costs \$12.75. Write an equation that could be used to find the cost of any number of admission tickets.  **$y = 12.75x$**



## Common Core Review

20. The costs of admission to a water park are shown in the table at the right. **7.RP.2b, 7.RP.2d, 8.EE.5**
- Find the constant rate of change between the quantities in the table. **The constant rate is 12.**
  - For this situation, what is the meaning of the constant rate of change? **It means the relationship is linear and that for each person the cost is \$12.**
  - What is the slope of a line that connects the ordered pairs for this relationship? **12**

Water Park Costs	
Number of People	Total Cost (\$)
$x$	$y$
3	36
4	48
5	60

Find the slope of the line that passes through each pair of points. **8.EE.5**

21.  $R(-1, 5)$  and  $S(0, 5)$  **0**  
 22.  $A(-2, 3)$  and  $B(-2, 8)$  **undefined**  
 23.  $C(6, 4)$  and  $D(4, 6)$  **-1**

Solve each problem using the percent equation. **7.RP.3**

24. 13.64 is what percent of 62? **22%**  
 25. 31.45 is 18.5% of what number? **170**



**Interactive Study Guide**

See page 203 for:  
 • Mid-Chapter Check