Lesson 4-6

Square Roots and Cube Roots

Interactive Study Guide

See pages 85–86 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

Why is it useful to write numbers in different ways?

Common Core State Standards

Content Standards
8.NS.2, 8.EE.2

Mathematical Practices
1, 2, 3, 4, 7

Vocabulary

square root
perfect square
radical sign
cube root
perfect cube

What You'll Learn

- Find square roots.
- Find cube roots.

Real-World Link

Rain Forest Tropical rainforests contain the greatest diversity of plants and animals on Earth—and they cover less than 5 percent of Earth's land! Just a four-square-mile patch of rainforest contains thousands of species of plants and trees, and hundreds of species of mammals, birds, reptiles, and amphibians.

Key Concept Square Roots

Words	A **square root** of a number is one of its two equal factors.
Symbols	If $x^2 = y$, then x is a square root of y.

Number like 9, 25, and 144 are **perfect squares**, because they are squares of integers. The opposite of squaring a number is finding the square root.

A **radical sign**, $\sqrt{}$ is used to indicate a nonnegative square root. Every positive number has both a positive and a negative square root.

$$\sqrt{36} = 6 \qquad -\sqrt{36} = -6 \qquad \pm\sqrt{36} = \pm 6 \text{ or } 6, -6$$

A negative number like -36 has no real-number square root because the square of a number cannot be negative. You will learn about real numbers in the next lesson.

Example 1

Tutor

Find each square root.

a. $\sqrt{9}$

$\sqrt{9} = 3$ Find the positive square root of 9; $3^2 = 9$.

b. $-\sqrt{64}$

$-\sqrt{64} = -8$ Find the negative square root of 64; $8^2 = 64$.

c. $\pm\sqrt{4}$

$\pm\sqrt{4} = \pm 2$ Find both square roots of 4; $2^2 = 4$.

d. $\sqrt{-81}$

There is no real square root because no number times itself is equal to -81.

Got It? Do these problems to find out.

1a. $\sqrt{49}$ **7**

1b. $-\sqrt{16}$ **-4**

1c. $\pm\sqrt{100}$ **± 10**

1d. $\sqrt{-49}$ **no real solution**

You can estimate the square root of an integer that is not a perfect square by determining between which two consecutive integers the square root lies.

Example 2

Tools Tutor

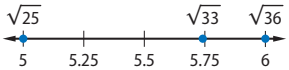
Estimate each square root to the nearest integer.

a. $\sqrt{33}$

The largest perfect square less than 33 is 25. $\sqrt{25} = 5$

The smallest perfect square greater than 33 is 36. $\sqrt{36} = 6$

Plot each square root on a number line. Then estimate $\sqrt{33}$.



$25 < 33 < 36$ Write an inequality.

$5^2 < 33 < 6^2$ $25 = 5^2$ and $36 = 6^2$

$\sqrt{5^2} < 33 < \sqrt{6^2}$ Find the square root of each number.

$5 < \sqrt{33} < 6$ Simplify.

So, $\sqrt{33}$ is between 5 and 6. Since 33 is closer to 36 than to 25, the best integer estimate for $\sqrt{33}$ is 6.

Check Check using a calculator.

[2nd] [√] 33 [ENTER] **5.744562647**

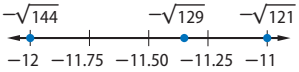
$\sqrt{33} \approx 6$ ✓

b. $-\sqrt{129}$

The largest perfect square less than 129 is 121. $\sqrt{121} = -11$

The smallest perfect square greater than 129 is 144. $\sqrt{144} = -12$

The negative square root of 129 is between the integers −11 and −12. Plot each square root on a number line. Then estimate $-\sqrt{129}$.



So, $-\sqrt{129}$ is between −12 and −11. Since 129 is closer to 121 than to 144, the best integer estimate for $-\sqrt{129}$ is −11.

Check Check using a calculator.

[(−)] [2nd] [√] 129 [ENTER] **−11.357816691**

$\sqrt{-129} \approx -11$ ✓

Watch Out!

When keying your calculator to find the negative of a square root, for example $-\sqrt{129}$, be sure to use the [(−)] button instead of the [−] button for the negative sign.

Got It? Do these problems to find out.

2a. $\sqrt{60}$ **8**

2b. $-\sqrt{23}$ **−5**

2c. $\sqrt{14}$ **4**

2d. $-\sqrt{79}$ **−9**

When finding square roots in real-world situations, use the positive, or *principal*, square root when a negative answer does not make sense.

Real World Example 3

Tutor

Choose a Form
Express a number as a square root if an exact answer is needed. Express a number as a decimal if an approximation is sufficient.

On a clear day, the number of miles a person can see to the horizon can be found using the formula $d = 1.22 \cdot \sqrt{h}$, where d is the distance to the horizon in miles and h is the person's distance from the ground in feet. The observation deck of Seattle's Space Needle is 520 feet high. How far to the horizon can a person standing on the observation deck see? Round to the nearest tenth.

Estimate The distance is between $1 \cdot \sqrt{400}$ and $1 \cdot \sqrt{900}$. So, it's between 20 and 30.

$d = 1.22 \cdot \sqrt{h}$	Write the equation.
$= 1.22 \cdot \sqrt{520}$	Replace h with 520.
$\approx 1.22 \cdot 22.8$	Use a calculator.
≈ 27.8	Simplify.

The approximate distance to the horizon is 27.8 miles to the nearest tenth.

Check for Reasonableness $20 < 27.8 < 30$ ✓

Got It? **Do these problems to find out.**

3a. Spring Port Ledge Lighthouse in Maine is approximately 55 feet tall. Calculate about how far a person who is standing at the top of the lightbouse can see on a clear day. Round to the nearest tenth of a mile. **9.0 mi**

3b. The observation deck of the Washington Monument is 500 feet high. Calculate about how far a person on the observation deck can see on a clear day. Round to the nearest tenth of a mile. **27.3 mi**

Key Concept Cube Roots

Words	A cube root of a number is one of its three equal factors.
Symbols	If $x^3 = y$, then $x = \sqrt[3]{y}$.
Examples	Since $2 \times 2 \times 2$ or $2^3 = 8$, 2 is a cube root of 8. Since $-6 \times (-6) \times (-6) = -216$, -6 is a cube root of -216.

A **cube root** of a number is one of three equal factors of the number. The symbol $\sqrt[3]{}$ is used to indicate the cube root of a number.

Every integer has exactly one cube root.

- The cube root of a positive number is positive.
- The cube root of zero is zero.
- The cube root of a negative number is negative.

Example 4

Tutor

Find each cube root.

a. $\sqrt[3]{343}$

$\sqrt[3]{343} = 7$ $\quad 7^3 = 7 \cdot 7 \cdot 7$ or 343

b. $\sqrt[3]{-729}$

$\sqrt[3]{-729} = -9$ $\quad (-9)^3 = (-9) \cdot (-9) \cdot (-9)$ or -729

Got It? Do these problems to find out.

4a. $\sqrt[3]{64}$ **4**

4b. $\sqrt[3]{-1331}$ **−11**

You can also estimate cube roots mentally by using **perfect cubes**.

Example 5

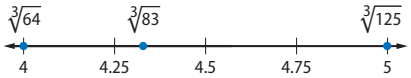
Tutor

Estimate $\sqrt[3]{83}$ to the nearest integer. Do not use a calculator.

$\sqrt[3]{83}$

The first perfect cube less than 83 is 64. $\quad \sqrt[3]{64} = 4$

The first perfect cube greater than 83 is 125. $\quad \sqrt[3]{125} = 5$



The cube root of 83 is between the integers 4 and 5. Since 83 is closer to 64 than to 125, you can expect $\sqrt[3]{83}$ to be closer to 4 than to 5.

Got It? Do these problems to find out.

5a. $\sqrt[3]{72}$ **4**

5b. $\sqrt[3]{-2024}$ **−13**

Guided Practice

Check

Find each square root. (Example 1)

1. $\sqrt{16}$ **4**

2. $-\sqrt{100}$ **−10**

3. $\pm\sqrt{81}$ **±9**

Estimate each square root to the nearest integer. (Example 2)

4. $\sqrt{27}$ **5**

5. $-\sqrt{48}$ **−7**

6. $\pm\sqrt{39}$ **±6**

7. A baseball diamond is actually a square with an area of 8100 square feet. Most baseball teams cover their diamond with a tarp to protect it from the rain. The sides are all the same length. How long is the tarp on each side? (Example 3) **90 ft**

Find each cube root. (Example 4)

8. $\sqrt[3]{512}$ **8**

9. $\sqrt[3]{2197}$ **13**

10. $\sqrt[3]{-1000}$ **−10**

11. $\sqrt[3]{-343}$ **−7**

Estimate each cube root to the nearest integer. (Example 5)

12. $\sqrt[3]{74}$ **4**

13. $\sqrt[3]{39}$ **3**

14. $\sqrt[3]{-636}$ **−9**

15. $\sqrt[3]{-879}$ **−10**