Lesson 2

Complex Fractions and Unit Rates

What You'll Learn

List two headings you would use to make an outline of the lesson.

- _____
- _____

Real-World Link

Speed Skating Dana is skating laps to train for a speed skating competition. She can skate 1 lap in 40 seconds.

1. Write a ratio in simplest form comparing Dana's time to her number of laps.

| Dana's time (s) | $\cdots\!\!\rightarrow$ | 40 |
| Number of Laps | $\cdots\!\!\rightarrow$ | 1 |

(handwritten) UNIT RATE (BECAUSE THE DENOMINATOR IS 1)

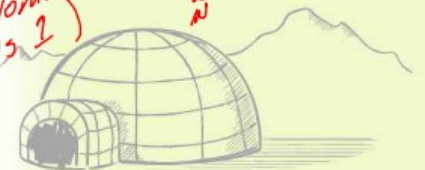*(handwritten)* NUMERATOR ↑ / DENOMINATOR ↓

2. Suppose Dana skates for 20 seconds. How many laps will she skate?

(handwritten) $\dfrac{20}{0.5} = \dfrac{20}{\frac{1}{2}}$

3. Write the ratio of Dana's time from Exercise 2 to her number of laps.

| Dana's time | $\cdots\!\!\rightarrow$ | 20 |
| Number of Laps | $\cdots\!\!\rightarrow$ | $\dfrac{1}{2}$ |

(handwritten) COMPLEX FRACTION

4. How could you simplify the ratio you wrote in Exercise 3?

_____

Page 1

$$\frac{1}{4} \div \frac{2}{1} = \frac{1}{4} \times \frac{1}{2}$$

$$\frac{3}{1} \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = \frac{12}{1} = 12$$

$$\frac{6}{1} \div \frac{1}{3} = \frac{6}{1} \times \frac{3}{1} = \frac{18}{1} = 18$$

Divide Fractions

To divide by a whole number, first write it as a fraction with a denominator of 1. Then multiply by the reciprocal.

So, $\frac{\frac{1}{4}}{2}$ can be written as $\frac{1}{4} \div \frac{2}{1}$.

$$\frac{3}{4} = 3 \div 4$$

$$\frac{2}{5} = 2 \div 5$$

$$\frac{19}{43} = 19 \div 43$$

a. _____

b. _____

c. _____

d. _____

Simplify a Complex Fraction

Fractions like $\frac{20}{\frac{1}{2}}$ are called complex fractions. **Complex fractions** are fractions with a numerator, denominator, or both that are also fractions. Complex fractions are simplified when both the numerator and denominator are integers.

$$\frac{1}{2} = 1 \div 2 = 0.5$$

Examples
Tutor

1. Simplify $\frac{\frac{1}{4}}{2}$.

Recall that a fraction can also be written as a division problem.

$$\frac{\frac{1}{4}}{2} = \frac{1}{4} \div 2 \qquad \text{Write the complex fraction as a division problem.}$$

$$= \frac{1}{4} \times \frac{1}{2} \qquad \text{Multiply by the reciprocal of 2, which is } \frac{1}{2}.$$

$$= \frac{1}{8} \qquad \text{Simplify.}$$

So, $\frac{\frac{1}{4}}{2}$ is equal to $\frac{1}{8}$.

2. Simplify $\frac{1}{\frac{1}{2}}$.

Write the fraction as a division problem.

$$\frac{1}{\frac{1}{2}} = 1 \div \frac{1}{2} \qquad \text{Write the complex fraction as a division problem.}$$

$$= \frac{1}{1} \times \frac{2}{1} \qquad \text{Multiply by the reciprocal of } \frac{1}{2}, \text{ which is } \frac{2}{1}.$$

$$= \frac{2}{1} \text{ or } 2 \qquad \text{Simplify.}$$

So, $\frac{1}{\frac{1}{2}}$ is equal to 2.

Show your work.

Got It? Do these problems to find out.

a. $\frac{\frac{2}{3}}{\frac{2}{3}}$ ← $2 \div \frac{2}{3} = \frac{2}{1} \times \frac{3}{2} = \frac{6}{2} = 3$

b. $\frac{6}{\frac{1}{3}}$ $\frac{6}{1} \div \frac{1}{3}$

c. $\frac{\frac{2}{3}}{\frac{7}{1}}$ ← $\frac{2}{3} \div \frac{7}{1} = \frac{2}{3} \cdot \frac{1}{7} = \frac{14}{3} = 4\frac{2}{3}$

d. $\frac{\frac{2}{4}}{2}$

$$\frac{2}{\frac{3}{7}} = 2 \div \frac{3}{7} = 2 \times \frac{7}{3} = \frac{14}{3}$$

Page 2

Find Unit Rates

When the fractions of a complex fractions represent different units, you can find the unit rate.

Real World Examples

Tutor

3. Josiah can jog $1\frac{1}{3}$ miles in $\frac{1}{4}$ hour. Find his average speed in miles per hour.

Write a rate that compares the number of miles to hours.

$$\frac{1\frac{1}{3}\text{ mi}}{\frac{1}{4}\text{ h}} = 1\frac{1}{3} \div \frac{1}{4}$$ Write the complex fraction as a division problem.

$$= \frac{4}{3} \div \frac{1}{4}$$ Write the mixed number as an improper fraction.

$$= \frac{4}{3} \times \frac{4}{1}$$ Multiply by the reciprocal of $\frac{1}{4}$, which is $\frac{4}{1}$.

$$= \frac{16}{3} \text{ or } 5\frac{1}{3}$$ Simplify.

So, Josiah jogs at an average speed of $5\frac{1}{3}$ miles per hour.

4. Tia is painting her house. She paints $34\frac{1}{2}$ square feet in $\frac{3}{4}$ hour. At this rate, how many square feet can she paint each hour?

Write a ratio that compares the number of square feet to hours.

$$\frac{34\frac{1}{2}\text{ ft}^2}{\frac{3}{4}\text{ h}} = 34\frac{1}{2} \div \frac{3}{4}$$ Write the complex fraction as a division problem.

$$= \frac{69}{2} \div \frac{3}{4}$$ Write the mixed number as an improper fraction.

$$= \frac{69}{2} \times \frac{4}{3}$$ Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{276}{6} \text{ or } 46$$ Simplify.

So, Tia can paint 46 square feet per hour.

Got It? Do these problems to find out.

Show your work.

e. Mr. Ito is spreading mulch in his yard. He spreads $4\frac{2}{3}$ square yards in 2 hours. How many square yards can he mulch per hour?

e. _____

f. Aubrey can walk $4\frac{1}{2}$ miles in $1\frac{1}{2}$ hours. Find her average speed in miles per hour.

f. _____

Example

5. On Javier's soccer team, about $33\frac{1}{3}\%$ of the players have scored a goal. Write $33\frac{1}{3}\%$ as a fraction in simplest form.

$$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100}$$ Definition of percent

$$= 33\frac{1}{3} \div 100$$ Write the complex fraction as a division problem.

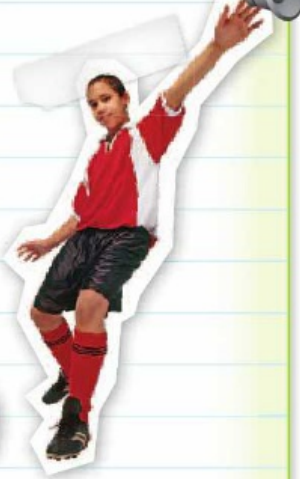$$= \frac{100}{3} \div 100$$ Write $33\frac{1}{3}$ as an improper fraction.

$$= \frac{\overset{1}{\cancel{100}}}{3} \times \frac{1}{\underset{1}{\cancel{100}}}$$ Multiply by the reciprocal of 100, which is $\frac{1}{100}$.

$$= \frac{1}{3}$$ Simplify.

So, about $\frac{1}{3}$ of Javier's team has scored a goal.

Guided Practice

Check ✓

Simplify. (Examples 1 and 2)

1. $\dfrac{18}{\frac{3}{4}} =$ _____

2. $\dfrac{\frac{3}{6}}{4} =$ _____

3. $\dfrac{\frac{1}{3}}{\frac{1}{4}} =$ _____

Show your work.

4. Pep Club members are making spirit buttons. They make 490 spirit buttons in $3\frac{1}{2}$ hours. Find the number of buttons the Pep Club makes per hour. (Examples 3 and 4) _____

5. A county sales tax is $6\frac{2}{3}\%$. Write the percent as a fraction in simplest form. (Example 5) _____

6. ⓔ **Building on the Essential Question** What is a complex fraction? _____
_____
_____
_____

Rate Yourself!

How confident are you about simplifying complex fractions? Check the box that applies.

☹   😐   🙂

☐ ☐ ☐ ☐ ☐

For more help, go online to access a Personal Tutor.

Tutor

Independent Practice

eHelp

Simplify. (Examples 1 and 2)

1. $\dfrac{\frac{1}{2}}{\frac{2}{3}} =$ _____

2. $\dfrac{2}{\frac{3}{11}} =$ _____

3. $\dfrac{\frac{8}{9}}{6} =$ _____

4. $\dfrac{\frac{2}{5}}{9} =$ _____

5. $\dfrac{\frac{4}{5}}{10} =$ _____

6. $\dfrac{\frac{1}{4}}{\frac{7}{10}} =$ _____

7 Mary is making pillows for her Life Skills class. She bought $2\frac{1}{2}$ yards of fabric. Her total cost was $15. What was the cost per yard? (Examples 3 and 4)

_____

8. Doug entered a canoe race. He rowed $3\frac{1}{2}$ miles in $\frac{1}{2}$ hour. What is his average speed in miles per hour? (Examples 3 and 4)

_____

9. Monica reads $7\frac{1}{2}$ pages of a mystery book in 9 minutes. What is her average reading rate in pages per minute? (Examples 3 and 4) _____

Write each percent as a fraction in simplest form. (Example 5)
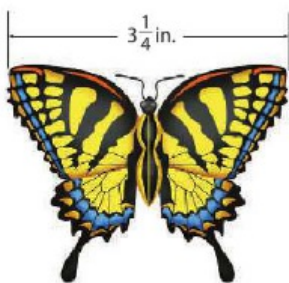
10. $56\frac{1}{4}\% =$ _____
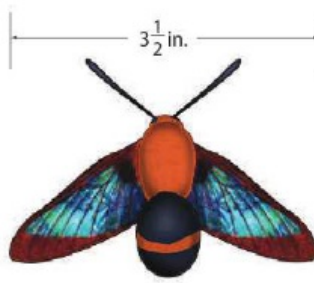
11. $15\frac{3}{5}\% =$ _____

12. $13\frac{1}{3}\% =$ _____

13. A bank is offering home loans at an interest rate of $5\frac{1}{2}\%$. Write the percent as a fraction in simplest form. (Example 5) _____

14. **CCSS** **Be Precise** Karl measured the wingspan of the butterfly and the moth shown below. How many times larger is the moth than the butterfly?

$3\frac{1}{4}$ in.

Black Swallowtail Butterfly

$3\frac{1}{2}$ in.

Hummingbird Moth

H.O.T. Problems Higher Order Thinking

15. **CCSS** **Construct an Argument** Explain how complex fractions can be used to solve problems involving ratios. _____

16. **CCSS** **Reason Inductively** Write three different complex fractions that simplify to $\frac{1}{4}$.

17. **CCSS** **Persevere with Problems** Use mental math to find the value of $\frac{15}{124} \cdot \frac{230}{30} \div \frac{230}{124}$.

18. **CCSS** **Justify Conclusions** The value of a mutual fund increased by $3\frac{1}{8}$%. Write $3\frac{1}{8}$% as a fraction in simplest form. Justify your answer.

19. **CCSS** **Persevere with Problems** The distance around the tire of a motorized scooter is 21.98 inches. The tires make one revolution every $\frac{1}{10}$ second. Find the speed of the scooter in miles per hour. Round to the nearest tenth. (*Hint: The speed of an object spinning in a circle is equal to the distance around the circle divided by the time it takes to complete one revolution.*)

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Extra Practice

Simplify.

20. $\dfrac{1}{\frac{1}{4}} =$ 4

$\dfrac{1}{\frac{1}{4}} = 1 \div \dfrac{1}{4}$

$= \dfrac{1}{1} \times \dfrac{4}{1}$

Homework Help →

$= \dfrac{4}{1}$ or 4

21. $\dfrac{12}{\frac{3}{5}} =$ _____

22. $\dfrac{\frac{9}{10}}{9} =$ _____

23. $\dfrac{\frac{1}{2}}{\frac{1}{4}} =$ _____

24. $\dfrac{\frac{1}{12}}{\frac{5}{6}} =$ _____

25. $\dfrac{\frac{5}{6}}{\frac{5}{9}} =$ _____

26. Mrs. Frasier is making costumes for the school play. Each costume requires 0.75 yard of fabric. She bought 6 yards of fabric. How many costumes can Mrs. Frasier make?

_____

27. A lawn company advertises that they can spread 7,500 square feet of grass seed in $2\frac{1}{2}$ hours. Find the number of square feet of grass seed that can be spread per hour.

_____

Write each percent as a fraction in simplest form.

28. $2\frac{2}{5}\% =$ _____

29. $7\frac{3}{4}\% =$ _____

30. $8\frac{1}{3}\% =$ _____

31. **CCSS** **Justify Conclusions** The value of a certain stock increased by $1\frac{1}{4}\%$. Explain how to write $1\frac{1}{4}\%$ as a fraction in simplest form. _____

_____

_____

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32. Debra bought $3\frac{1}{4}$ yards of fabric at a remnant sale for $13. Determine if each of the following remnant deals have the same unit price as Debra's deal. Select yes or no.

a. $4\frac{2}{3}$ yards for $16 ☐ Yes ☐ No

b. $2\frac{3}{4}$ yards for $11 ☐ Yes ☐ No

c. $6\frac{1}{2}$ yards for $26 ☐ Yes ☐ No

33. The table shows the distances traveled by 4 cyclists. Sort the speeds of the riders, in miles per hour, from slowest to fastest.

Bicycle Rides		
Rider	**Distance**	**Time**
Elena	$20\frac{1}{2}$ mi	$2\frac{1}{4}$ h
Julio	$12\frac{1}{4}$ mi	$1\frac{1}{2}$ h
Kevin	$20\frac{2}{3}$ mi	$1\frac{2}{3}$ h
Lorena	$33\frac{1}{4}$ mi	$2\frac{1}{3}$ h

	Rider	Speed (mph)
Slowest		
Fastest		

Which rider had the fastest rate of speed? ☐

Common Core Spiral Review

Fill in each box with the equivalent customary measurement. 5.MD.1

34. 2 feet = ☐ inches

35. 5 tons = ☐ pounds

36. 8 gallons = ☐ quarts

Fill in each box with the equivalent metric measurement. 5.MD.1

37. 1 meter = ☐ centimeters

38. 1 liter = ☐ millileters

39. 1 kilogram = ☐ grams