UPS charges $7 for the first pound, and $0.20 for each additional pound. FedEx charges $5 for the first pound and $0.30 for each additional pound. How many pounds, p, will it take for UPS and FedEx to cost the same? $p = 20$

GUESS AND CHECK

POUNDS	UPS	FED EX
0	7	5
5	8	6.50
10		

UPS $7 + 0.20p$ = FED EX $5 + 0.30p$ (1)

$-0.20p \quad -0.20p$

$7 = 5 + 0.10p$ (2)

$-5 \quad -5$

$2 = 0.10p$

$\frac{2}{0.10} = \frac{0.10p}{0.10}$

$20 = p$

$5 = 7 + 0.10x$

$-5 \quad -5$

$0 = 2 + 0.10x$

Lesson 8-5

Solving Equations with Variables on Each Side

Interactive Study Guide

See pages 179–180 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

How are equations and inequalities used to describe and solve multi-step problems?

Common Core State Standards

Content Standards
7.EE.4, 7.EE.4a, 8.EE.7, 8.EE.7b

Mathematical Practices
1, 3, 4, 7

What You'll Learn

- Solve equations with variables on each side.

Real-World Link

Camping Camping is an activity that many families like to do together. Many campsites offer rentals for equipment, like kayaks and bicycles. The table shows the rental fees for a certain campground.

Item	Deposit ($)	Cost per Day ($)
bicycle	3.00	5.50
kayak	6.00	5.00

Equations with Variables on Each Side

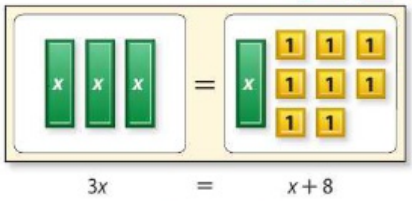
The equation $5.5x + 3 = 5x + 6$ can be used to find the number of days for which renting a bike costs the same as renting a kayak. To solve equations with variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variables on one side. Then solve the equation.

Example 1

Tools Tutor

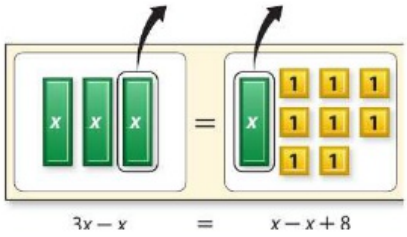
Solve $3x = x + 8$.

$3x = x + 8$ Write the equation.



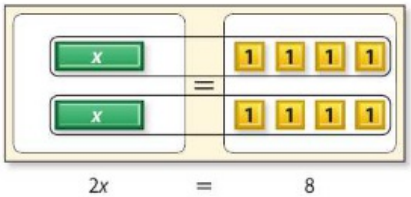
$3x = x + 8$
$-x = -x$
$2x = 8$

Since $3x$ is already alone on one side, use the Subtraction Property of to subtract x from each side. Simplify.



$\frac{2x}{2} = \frac{8}{2}$ Use the Division Property of Equality to divide each side by 2.

$x = 4$ Simplify.



The solution is 4.

Got It? Do these problems to find out.

1a. $7x = 5x + 4$

$-5x \quad -5x$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

$7(2) = 5(2) + 4$

$14 = 10 + 4$

$14 = 14$ ✓

1b. $3x - 2 = x$

$-3x \quad -3x$

$\frac{-2}{-2} = \frac{-2x}{-2}$

$1 = x$

$3(1) - 2 = 1$

$3 - 2 = 1$

Solve a Simpler Equation

Notice that the equations in the third step of each solution are similar to the equations that you solved in Lesson 8-2.

Example 2

Tutor

Solve $\frac{6}{5}y + 8 = \frac{4}{5}y - 10$.

$\frac{6}{5}y + 8 = \frac{4}{5}y - 10$	Write the equation.
$\frac{6}{5}y - \frac{4}{5}y + 8 = \frac{4}{5}y - \frac{4}{5}y - 10$	Subtraction Property of Equality
$\frac{2}{5}y + 8 = -10$	Simplify.
$\frac{2}{5}y + 8 - 8 = -10 - 8$	Subtraction Property of Equality
$\frac{2}{5}y = -18$	Simplify.
$\frac{5}{2} \cdot \frac{2}{5}y = \frac{5}{2} \cdot (-18)$	Multiplication Property of Equality
$y = -45$	Simplify.

Got It? Do these problems to find out.

2a. $2.1x + 3 = 3.1x - 2$

2b. $\frac{1}{2}p - 15 = \frac{3}{4}p - 3$

2a. $2.1x + 3 = 3.1x - 2$

$-2.1x \quad -2.1x$ ①

$3 = 1x - 2$

$+2 \quad +2$ ②

$\frac{5}{1} = \frac{1x}{1}$

$5 = x$

$2.1(5) + 3 = 3.1(5) - 2$

$10.5 + 3 = 15.5 - 2$

$13.5 \quad 13.5$ ✓

2b. $\frac{1}{2}p - 15 = \frac{3}{4}p - 3$

$\frac{2}{4}p - 15 = \frac{3}{4}p - 3$

$-\frac{2}{4}p \quad -\frac{2}{4}p$

$-15 = \frac{1}{4}p - 3$

$+3 \quad +3$

$\frac{4}{1}(-12) = (\frac{1}{4}p)\frac{4}{1}$

$-48 = p$

$\frac{1}{2}(-48) - 15 = \frac{3}{4}(-48) - 3$

$-24 - 15 = -36 - 3$

$-39 = -39$ ✓

Solve Verbal Problems

In some real-world situations you are asked to determine when the cost of two different products or services will be equal. This often results in an equation with variables on each side.

Example 3

Tutor

A personal trainer charges a one-time fee of $60 plus $25 for each individual session. A fitness club charges a yearly fee of $450 plus $10 for each session with a personal trainer. Write and solve an equation to determine for what number of sessions the costs will be equal.

Let s = number of sessions.

$60 + 25s = 450 + 10s$	Write the equation.
$60 + 25s - 10s = 450 + 10s - 10s$	Subtraction Property of Equality.
$60 + 15s = 450$	Simplify.
$60 - 60 + 15s = 450 - 60$	Subtraction Property of Equality.
$15s = 390$	Simplify.
$\frac{15s}{15} = \frac{390}{15}$	Division Property of Equality.
$s = 26$	Simplify.

You would need to have 26 training sessions in order for the costs to be equal.

Got It? Do this problem to find out.

3. Red Bird Cruises charges $85 per day plus a one-time fee of $75. King Cruises charges $100 per day plus a fee of $30. Write and solve an equation to determine for what number of days the charge for the cruises will be the same.

Guided Practice

Solve each equation. Check your solution. (Examples 1 and 2)

1. $x + 6 = 3x$
2. $4y = 2y - 10$
3. $1.4z - 6 = 2.9z + 9$
4. $-3.8t + 4 = 4.4t - 37$
5. $\frac{4}{9}x - 1 = \frac{5}{9}x + 2$
6. $\frac{3}{8}p - 3 = \frac{1}{8}p - 5$

7. An Internet movie rental company charges a yearly membership fee of \$50 plus \$1.99 per DVD rental. Your neighborhood rental store has no membership fee and charges \$3.99 per DVD rental. Write and solve an equation to find the number of DVDs for which the cost of each will be the same. (Example 3)

Independent Practice

Go online for Step-by-Step Solutions

Solve each equation. Check your solution. (Examples 1 and 2)

8. $2x + 3 = x$
9. $8 - v = 7v$
10. $3.2 + 0.3x = 0.2x + 1.4$
11. $0.4x = 2x + 1.2$
12. $7.2 - 3c = 2c - 2$
13. $3 - 3.7b = 10.3b + 10$
14. $-\frac{1}{4}x + 6 = \frac{2}{3}x + 28$
15. $\frac{2}{5}x - 8 = 20 + \frac{3}{4}x$

16. Use the table at the right to write and solve an equation to find the number of miles a rental car must be driven for each option to cost the same for one day. (Example 3)

ABC Auto Rental		
Option	Cost per Day	Cost per Mile
A	\$25	\$0.45
B	\$40	\$0.25

17. Denzel is comparing Web sites for downloading music. One charges a \$5 membership fee plus \$0.50 per song. Another charges \$1.00 per song but has no membership fee. Write and solve an equation to find how many songs Denzel would have to buy to spend the same amount at both Web sites. (Example 3)

Solve each equation. Check your solution.

18. $5.3x + 2 - 4.1x = 3.6x - 1.6$
19. $-2.2 + 0.3z = 3 - 0.5z - 0.8z$
20. $\frac{2}{5}c + \frac{4}{5}c - 6 = c + 7$
21. $\frac{1}{8}m + \frac{2}{5} + \frac{1}{2}m = \frac{3}{8}m + \frac{3}{4}m$
22. $-\frac{1}{3}x + \frac{5}{6}x + 2 = -\frac{1}{2} + \frac{1}{6}x$
23. $\frac{4}{5}c - \frac{1}{10}c = \frac{9}{10}c$

24. Gabriella bought some school supplies for \$48, a jacket for \$56, and then bought 3 CDs. Min did not buy any school supplies but bought 11 CDs. All the CDs cost the same amount, and both students spent the same amount of money. Write and solve an equation to find the cost of one CD.

25. **Financial Literacy** One cell phone company charges \$19.95 a month and a \$2.15 tax plus \$0.21 per text message, and a second company charges \$24.95 per month plus \$0.16 per text message. For how many text messages is the cost of the plans the same?

$-2.2 + 0.3z = 3 - 0.5z - 0.8z$

$-2.2 + 0.3z = 3 - 1.3z$

$+1.3z \quad +1.3z$

$1.3z$

$-2.2 + 1.6z = 3$

$+2.2 \quad +2.2$

$\frac{1.6z}{1.6} = \frac{5.2}{1.6}$

$z = \frac{52}{16}$

$3\frac{4}{16} = 3\frac{1}{4} = 3.25$

$\frac{4}{5}c - \frac{1}{10}c = \frac{9}{10}c \rightarrow \frac{8}{10}c - \frac{1}{10}c = \frac{9}{10}c$

$\frac{7}{10}c = \frac{9}{10}c$

Financial Literacy One cell phone company charges $19.95 a month and a $2.15 tax plus $0.21 per text message, and a second company charges $24.95 per month plus $0.16 per text message. For how many text messages is the cost of the plans the same?

$4 \quad 1 \quad \frac{9}{10} \quad \frac{8}{10} \quad \frac{1}{10}c = \frac{9}{10}c$

$19.95 + 2.15 + 0.21x = 24.95 + 0.16x$

$22.10 + 0.21x = 24.95 + 0.16x$

$-22.10 \qquad -22.10$

$0.21x = 2.85 + 0.16x$

$-0.16x \qquad -0.16x$

$\frac{0.05x}{0.05} = \frac{2.85}{0.05}$

$x = 57$

$19.95 + 2.15 + 0.21(57) = \$34.07$

$24.95 + 0.16(57) = \$34.07$

26. Five years ago Ang was $\frac{1}{2}$ as old as Paula. Now he is $\frac{3}{5}$ as old as Paula.

Complete the table shown below. Then use the table to write and solve an equation to find their current ages.

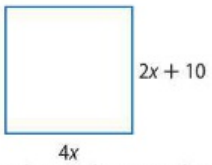
Student	5 years ago	Now
Ang	$\frac{1}{2}a$	■
Paula	a	$a+5$

27 Florida's coastline is 118 miles shorter than four times the coastline of Texas. It is also 983 miles longer than the coastline of Texas. Find the lengths of the coastlines of Florida and Texas.

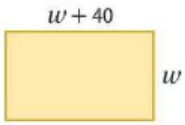
28. Use the square shown at the right.

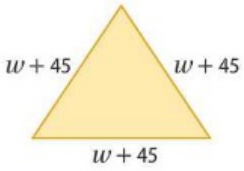
a. What is the value of x?

b. Find the length of each side of the square.



29. CCSS **Justify Conclusions** Jamie is going to fence the rectangular and triangular sections of grass shown below. The perimeters of the two sections are now equal. If w represents the width of the rectangle, how could you find the lengths of the sides of the rectangle and of the triangle? Justify your response and use your method to solve the problem.





H.O.T. Problems Higher Order Thinking

30. CCSS **Identify Structure** Write an equation that has variables on each side and has a solution of -2.

31. CCSS **Persevere with Problems** The formula $F = 1.8C + 32$ can be used to find the temperature in degrees Fahrenheit F when the temperature is given in degrees Celsius C. For what value is the temperature in degrees Fahrenheit equal to the temperature in degrees Celsius? Justify your reasoning by writing and solving an equation. (*Hint:* If Fahrenheit and Celsius are equal, they can be assigned the same variable.)

32. CCSS **Find the Error** Mykia is solving the equation $10x + 6 = 8x - 4$. Find her mistake and correct it.

$$10x + 6 = 8x - 4$$
$$10x - 10x + 6 = 8x - 4 - 10x$$
$$6 = 4 - 2x$$
$$2 = -2x$$
$$-1 = x$$

33. **Building on the Essential Question** Write a real-world problem that could be solved by using the equation $54 + 3.5x = 8x$. Then solve the equation and interpret your solution.

Standardized Test Practice

34. Yesterday, the math club had 1 less than 3 times their average attendance. Last week they had 3 more than their average attendance. If the attendance for both weeks was equal, what is the average attendance?

A 1 **C** 3
B 2 **D** 4

35. For which of the following is -8 a solution?

F $-2c + 18 = 10c + 12$
G $6m - 15 = 9m + 9$
H $4 + 7s = 5s + 20$
J $5d - 13 = 19 - 3d$

36. Short Response What is the solution of the equation $12x + 4 = 2x - 16$?

37. A cellular company has the following options for text messaging plans.

Text Plans	Monthly Fee	Cost per Message
Plan A	$10	$0.15
Plan B	$20	$0.05

Which equation shows how many text messages would need to be sent in order for the costs for one month to be the same?

A $10 + 0.05m = 20 + 0.15m$
B $10m + 0.15 = 20m + 0.05$
C $10 + 0.15m = 20 + 0.05m$
D $10(m + 0.15) = 20(m + 0.05)$

Common Core Review

Solve each problem by writing and solving an equation. 7.EE.4a

38. Carla paid $45 to join a golf camp for the summer. She will also pay $15 for every private lesson that she takes. If she has budgeted $225 for the camp, how many private lessons can she take?

39. An accountant charges $22.50 plus $17.50 per hour for a consultation. The Chen family paid $83.75 for a tax consultation. How long did their consultation last?

40. STEM An atom of chlorine has 36 fewer protons than an atom of iodine. Together, an atom of chlorine and an atom of iodine have a total of 70 protons. How many protons does an atom of iodine have?

Simplify each expression. 7.EE.1

41. $2x + 5x$ **42.** $7b + 2b$ **43.** $y + 10y$
44. $5m + 4 + 7m$ **45.** $6s + 10 - 4s - 3$ **46.** $\frac{2}{3}n + \frac{2}{3} + \frac{1}{3}n$
47. $3y - 2(y + 1)$ **48.** $0.2p + 0.5(2p + 7)$ **49.** $-2(x - 2) + 4x$

50. The table shows the number of tornadoes that occurred in Nebraska in June and the total number of tornadoes for selected years. 7.EE.3

a. What decimal part, rounded to the nearest hundredth, of the annual tornadoes occurred in June for each year?

b. In which years did more than two-fifths of the tornadoes occur in June?

Annual Nebraska Tornadoes		
Year	June	Total
2009	24	39
2006	8	22
2003	43	81
2000	21	61

Evaluate each expression. 6.EE.2c

51. $8c + 5$, if $c = 6$ **52.** $22 - 3h$, if $h = 4$ **53.** $36 - (-6g)$, if $g = -2$

$1.4z - 6 = 2.9z + 9$

$-1.4z \qquad -1.4z$

$-6 = 1.5z + 9$

$-9 \qquad -9$

$\frac{-15}{1.5} = \frac{1.5z}{1.5}$

$-10 = z$

$1.4z - 6 = 2.9z + 9$

$+6 \qquad +6$

$1.4z = 2.9z + 15$

$-2.9z \qquad -2.9z$

$\frac{-1.5z}{-1.5} = \frac{15}{-1.5}$

$z = -10$

$1.4z - 6 = 2.9z + 9$

$-2.9z \qquad -2.9z$

$-1.5z + 6 = 9$

$+6 \qquad +6$

$\frac{-1.5z}{-1.5} = \frac{15}{-1.5}$

$z = -10$