Lesson 8-7

Solving Inequalities

Interactive Study Guide

See pages 183–184 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

How are equations and inequalities used to describe and solve multi-step problems?

Common Core State Standards

Content Standards
7.EE.4, 7.EE.4b

Mathematical Practices
1, 3, 4

What You'll Learn

- Solve inequalities by using the Addition and Subtraction Properties of Inequality.
- Solve inequalities by multiplying or dividing by a positive or negative number.

Real-World Link

Pets Did you know that 39 percent of U.S. households own at least one dog? The amount of food that you feed your dog should be based on the dog's weight. For example, an adult dog that weighs 10 pounds or less needs about $\frac{3}{4}$ cup of food per day. A dog that weighs more than 50 pounds needs 2 to 4 cups of food each day.

Key Concept Addition and Subtraction Properties

Words When you add or subtract the same number from each side of an inequality, the inequality remains true.

Symbols For all numbers a, b, and c,

1. If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
2. If $a > b$, then $a + c > b + c$ and $a - c > b - c$.

Examples

$5 < 9$
$5 + 4 < 9 + 4$
$9 < 13$

$11 > 6$
$11 - 3 > 6 - 3$
$8 > 3$

The above properties are useful for solving inequalities. Note that these properties are also true for $a \leq b$ and $a \geq b$.

Example 1

Tutor

Solve $x + 5 > 12$. Check your solution.

$x + 5 > 12$ Write the inequality.
$-5 \quad -5$ Subtraction Property of Inequality
$x > 7$ Simplify.

Check
$x + 5 > 12$ Write the inequality.
$9 + 5 \stackrel{?}{>} 12$ Replace x with any number greater than 7.
$14 > 12$ ✓ The statement is true.

Got It? Do these problems to find out.

1a. $y + 10 < 3$ $y < -7$

1b. $x + 7 \geq 10$ $x \geq 3$

Equations and Inequalities

In an equation, if $a = b$, then $b = a$. In an inequality, if $a < b$, then $b > a$.

Example: $7 = 2 + 5$ and $2 + 5 = 7$
$2 + 8 > 7$
but $7 < 2 + 8$

When graphing inequalities, it is often easier to visualize the solution when the variable is on the left side of the inequality symbol.

Example 2

Tools Tutor

Solve $3 \le b - 1\frac{1}{3}$. Graph the solution on a number line.

$3 \le b - 1\frac{1}{3}$ Write the inequality.

$3 + 1\frac{1}{3} \le b - 1\frac{1}{3} + 1\frac{1}{3}$ Addition Property of Inequality

$4\frac{1}{3} \le b$ or $b \ge 4\frac{1}{3}$ Simplify.

The solution is $b \ge 4\frac{1}{3}$.

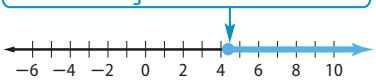
Check $3 \le b - \frac{1}{3}$ Write the inequality.

$3 \stackrel{?}{\le} 4\frac{1}{3} - 1\frac{1}{3}$ Replace b with $4\frac{1}{3}$.

$3 \le 3$ ✓ The statement is true.

Graph the solution.

Since the inequality symbol is $\ge$, draw a closed dot at $4\frac{1}{3}$ with an arrow to the right.



Got It? Do these problems to find out.

Solve each equation. Graph the solution on a number line.

2a. $3 \ge g + 7$ $g \le -4$

2b. $b + \frac{5}{7} > 2$ $b > 1\frac{2}{7}$

2a–2b. See Answer Appendix for number lines.

Positive Number

The statement $c > 0$ means that c is a positive number.

Key Concept Multiplication and Division Properties

Words When you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Symbols For all numbers a, b, and c, where $c > 0$,

1. If $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
2. If $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Examples

$-6 < 10$ $\quad$ $20 > 16$

$-6 \cdot 2 < 10 \cdot 2$ $\quad$ $\frac{20}{4} > \frac{16}{4}$

$-12 < 20$ $\quad$ $5 > 4$

These properties are also true for $a \le b$ and $a \ge b$.

Some inequalities, like $4x > 8$, are solved by multiplication or division. You can multiply or divide each side of an inequality by a positive number and the inequality is still true.

Example 3

Words to Symbols
Remember, *at most* translates to ≤, while *at least* translates to ≥.

Macy is making each of her 7 friends a bracelet. In addition to charms, she needs one package of wire for all of the bracelets. The wire is $6. She does not want to spend more than $45 on the bracelets. Find the maximum cost for each bracelet.

Since Macy wants to spend at most $45, write and solve an inequality using the symbol ≤.

Words	Number of friends times cost of each bracelet plus $6 must be less than or equal to $45.
Variable	Let c = the cost of each bracelet.
Inequality	$7c + \$6 \leq \$45$

$7c + 6 \leq 45$	Write the inequality.
$7c + 6 - 6 \leq 45 - 6$	Subtraction Property of Inequality
$7c \leq 39$	Simplify.
$\frac{7c}{7} \leq \frac{39}{7}$	Division Property of Inequality
$c \leq 5\frac{4}{7}$ or $5.\overline{571428}$	Simplify.

Macy can spend no more than $5.57 per bracelet.

Got It? Do these problems to find out.

3a. Alfonzo works for a lawn service company. It takes Alfonzo $\frac{3}{4}$ hour to mow a lawn. If he works more than 8 hours, he gets a $\frac{1}{2}$ hour lunch. Write and solve an inequality to find the number of lawns he can mow if he works at least 14 hours.

3a. $\frac{3}{4}n + \frac{1}{2} \geq 14$; $n \geq 18$ lawns

3b. Keiko prepared 28 bags of granola to sell at a school fundraiser. She also received a $10 donation. Write and solve an inequality to find the price she should charge for each bag of granola if she wants to raise at least $115.

3b. $28p + 10 \geq 115$; $p \geq \$3.75$

Negative Number
The statement $c < 0$ means that c is a negative number.

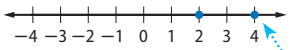
Key Concept Multiplication and Division Properties

Words	When you multiply or divide each side of an inequality by the same *negative* number, the inequality symbol must be reversed for the inequality to remain true.
Symbols	For all numbers a, b, and c, where $c < 0$, 1. If $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. 2. If $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
Examples	$-4 < 5$ $\quad$ $18 > -12$ $-4 \cdot (-3) > 5 \cdot (-3)$ $\quad$ $\frac{18}{-3} < \frac{-12}{-3}$ $12 > -15$ $\quad$ $-6 < 4$

These properties are also true for $a \leq b$ and $a \geq b$.

To understand the properties shown on the previous page, consider what happens when each side of an inequality is multiplied or divided by a negative number.

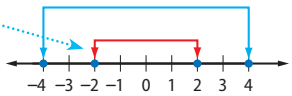
Graph 2 and 4 on a number line.



Since 4 is to the right of 2, $2 < 4$.

Now, multiply each number by −1.

Since −2 is to the right of −4, $-2 > -4$.



Notice that the numbers being compared switched positions as a result of being multiplied by a negative number. In other words, their order reversed. This suggests the properties shown on the previous page.

Example 4

Tools Tutor

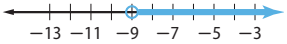
Solve each inequality. Then graph the solution on a number line.

a. $-5x < 45$

$-5x < 45$ Write the inequality.

$\frac{-5x}{-5} > \frac{45}{-5}$ Division Property of Inequality

$x > -9$ Simplify.



b. $\frac{b}{-8} - 2 \geq -8$

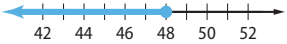
$\frac{b}{-8} - 2 \geq -8$ Write the inequality.

$\frac{b}{-8} - 2 + 2 \geq -8 + 2$ Addition Property of Inequality

$\frac{b}{-8} \geq -6$ Simplify.

$(-8)\frac{b}{-8} \leq -6(-8)$ Multiplication Property of Inequality

$b \leq 48$ Simplify.



Checking Solutions

For Example 4a, try a number that is less than −9 to show it is *not* a solution.

$-5x < 45$

$-5(-10) \stackrel{?}{<} 45$

$50 \not< 45$

Got It? Do these problems to find out.

4a–4b. See Answer Appendix for number lines.

4a. $-\frac{y}{4} < 3$ $y > -12$

4b. $7 \geq -2f$ $f \geq -3.5$

Guided Practice

Check

Solve each inequality. Check your solutions. (Example 1)

1. $y + 7 \leq 12$ $y \leq 5$

2. $b + 20 > -13$ $b > -33$

3. $-7 < x + (-3)$ $x > -4$

Solve each inequality. Graph each solution on a number line. (Examples 2 and 4) **See Answer Appendix for number lines.**

4. $d - 9.3 \geq 12.5$ $d \geq 21.8$

5. $3\frac{1}{5} > f - \frac{4}{5}$ $f < 4$

6. $g - 22 \leq -40$ $g \leq -18$

7. $-12 \geq -3q - 18$ $q \geq -2$

8. $-8z \leq -24$ $z \geq 3$

9. $18 > -\frac{2}{3}g$ $g > -27$

10. Isabel earns $50 plus $2.50 for each table she cleans. Write and solve an inequality to find how many tables she must clean to earn at least $120. (Example 3) $2.5t + 50 \geq 120$; at least 28 tables