Pg 65

Lesson 7

Constant Rate of Change

What You'll Learn

Scan the text on the following two pages. Write two facts you learned about constant rate of change.

- _____
- _____

Essential Question

HOW can you show that two objects are proportional?

Vocabulary

rate of change
constant rate of change

Common Core State Standards

Content Standards
7.RP.2, 7.RP.2b, 7.RP.2d

Mathematical Practices
1, 3, 4

Vocabulary Start-Up

A **rate of change** is a rate that describes how one quantity changes in relation to another. In a linear relationship, the rate of change between any two quantities is the same. A linear relationship has a **constant rate of change**.

Real-World Link

A computer programmer charges customers per line of code written. Fill in the blanks with the amount of change between consecutive numbers.

Lines of Code	0	50	100	150	200
Cost ($)	0	1,000	2,000	3,000	4,000

50 50 50 50

1000 1000 1000 1000

÷50 ÷50

$\frac{1000}{50} = \frac{20}{1}$ ← UNIT RATE

$\frac{3000}{150} = \frac{20}{1}$

$\frac{4000}{200} = \frac{20}{1}$

Label the diagram below with the terms *change in lines*, *change in dollars*, and *constant rate of change*.

$$\frac{\text{CHANGE IN DOLLARS}}{\text{CHANGE IN LINES}} = \frac{\$1{,}000}{50 \text{ lines}} = \frac{\$20}{1 \text{ line}} \text{ unit rate}$$

The CONSTANT RATE OF CHANGE is $20 per line of programming code.

Work Zone

Use a Table

You can use a table to find a constant rate of change.

Example

Watch Tutor

1. **The table shows the amount of money a booster club makes washing cars for a fundraiser. Use the information to find the constant rate of change in dollars per car.**

Cars Washed	
Number	**Money ($)**
5	40
10	80
15	120
20	160

+5, +5, +5 (Number) +40, +40, +40 (Money)

$\frac{40}{5} = \frac{8}{1}$

$\frac{160}{20} = \frac{8}{1}$

Unit Rate
A rate of change is usually expressed as a unit rate.

Find the unit rate to determine the constant rate of change.

$$\frac{\text{change in money}}{\text{change in cars}} = \frac{40 \text{ dollars}}{5 \text{ cars}}$$ The money earned increases by $40 for every 5 cars.

$$= \frac{8 \text{ dollars}}{1 \text{ car}}$$ Write as a unit rate.

So, the number of dollars earned increases by $8 for every car washed.

Got It? Do these problems to find out.

a. The table shows the number of miles a plane traveled while in flight. Use the information to find the approximate constant rate of change in miles per minute.

Time (min)	30	60	90	120
Distance (mi)	290	580	870	1,160

Show your work.

a. _____

b. The table shows the number of students that buses can transport. Use the table to find the constant rate of change in students per school bus.

Number of Buses	2	3	4	5
Number of Students	144	216	288	360

1, 1, 1 (Number of Buses) 72, 72, 72 (Number of Students)

$\frac{72 \text{ students}}{1 \text{ bus}}$ "PER"

b. _____

Use a Graph

You can also use a graph to find a constant rate of change and to analyze points on the graph.

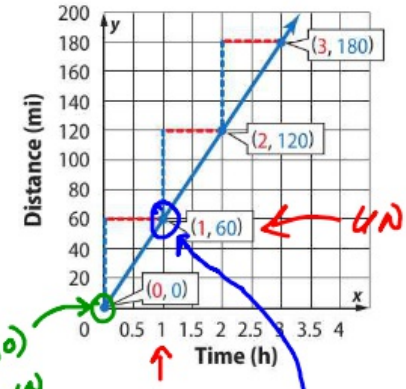
Examples

Tutor

2. **The graph represents the distance traveled while driving on a highway. Find the constant rate of change.**

To find the rate of change, pick any two points on the line, such as (0, 0) and (1, 60).

$$\frac{\text{change in miles}}{\text{change in hours}} = \frac{(60 - 0)\text{ miles}}{(1 - 0)\text{ hours}} = \frac{60\text{ miles}}{1\text{ hour}}$$



(0,0) ZERO MILES IN ZERO HOURS

UNIT RATE

60 MILES IN 1 HOUR

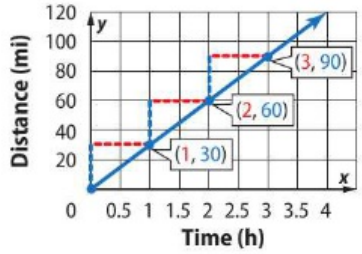
Ordered Pairs
The ordered pair (2, 120) represents traveling 120 miles in 2 hours.

3. **Explain what the points (0, 0) and (1, 60) represent.**

The point (0, 0) represents traveling zero miles in zero hours. The point (1, 60) represents traveling 60 miles in 1 hour. Notice that this is the constant rate of change.

Got It? **Do these problems to find out.**

c. Use the graph to find the constant rate of change in miles per hour while driving in the city.



Show your work.

c. _____

d. On the lines below, explain what the points (0, 0) and (1, 30) represent.

$\frac{30}{1}$, $\frac{60}{2} = \frac{30}{1}$, $\frac{90}{3} = \frac{30}{1}$

THE CONSTANT RATE OF CHANGE = $\frac{30}{1} = 30$

(0,0) ZERO MILES IN ZERO HOURS

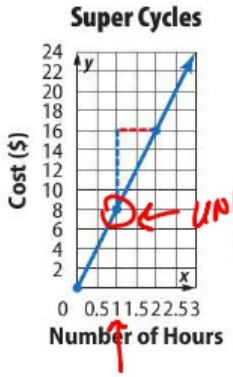
(1, 30) 30 MILES IN ONE HOUR

Example

4. **The table and graph below show the hourly charge to rent a bicycle at two different stores. Which store charges more per bicycle? Explain.**

Pedals Rentals	
Time (hour)	**Cost ($)**
2	24
3	36
4	48

+1, +1, +12, +12



The cost at Pedals Rentals increases by $12 every hour. The cost at Super Cycles increases by $8 every hour.

So, Pedals Rentals charges more per hour to rent a bicycle.

Guided Practice

1. The table and graph below show the amount of money Mi-Ling and Daniel save each week. Who saves more each week? Explain. (Examples 1, 2, and 4)

Mi-Ling's Savings	
Time (weeks)	**Savings ($)**
2	$30
3	$45
4	$60

2. Refer to the graph in Exercise 1. Explain what the points (0, 0) and (1, 10) represent. (Example 3)

3. **Building on the Essential Question** How can you find the unit rate on a graph that goes through the origin?

Rate Yourself!

Are you ready to move on? Shade the section that applies.

For more help, go online to access a Personal Tutor.

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions eHelp

Find the constant rate of change for each table. (Example 1)

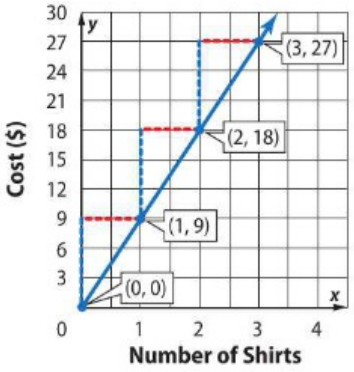
1.

Time (s)	Distance (m)
1	6
2	12
3	18
4	24

2.

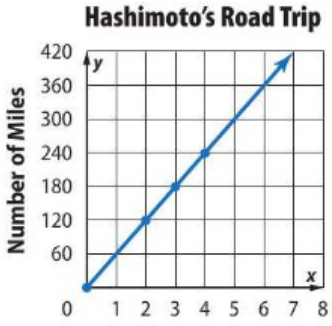
Items	Cost ($)
2	18
4	36
6	54
8	72

3. The graph shows the cost of purchasing T-shirts. Find the constant rate of change for the graph. Then explain what points (0, 0) and (1, 9) represent. (Examples 2 and 3)



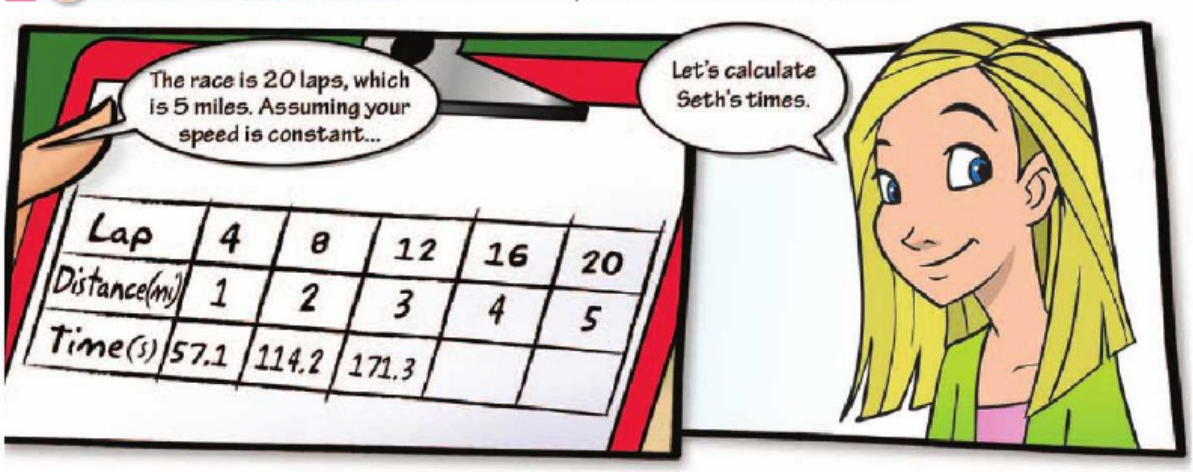
4. The Guzman and Hashimoto families each took a 4-hour road trip. The distances traveled by each family are shown in the table and graph below. Which family averaged fewer miles per hour? Explain. (Example 4)

Guzman's Road Trip	
Time (hours)	Distance (miles)
2	90
3	135
4	180



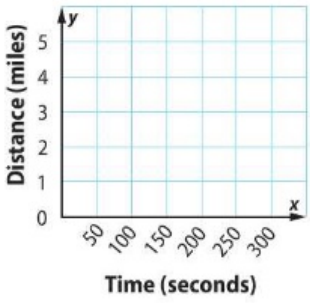
5. At 1:00 P.M., the water level in a pool is 13 inches. At 1:30 P.M., the water level is 18 inches. At 2:30 P.M., the water level is 28 inches. What is the constant rate of change?

6 **Model with Mathematics** Refer to the lap times for Exercises **a** and **b**.



a. How long does it take Seth to race 1 mile? Write the constant rate of change in miles per second. Round to the nearest hundredth. _____

b. Graph the ordered pairs (time, distance) on the coordinate plane at the right. Connect the points with a solid line.



H.O.T. Problems

7. **Model with Mathematics** Make a table where the constant rate of change is 6 inches for every foot.

Feet	Inches

8. **Justify Conclusions** The terms in sequence A increase by 3. The terms in sequence B increase by 8. In which sequence do the terms form a steeper line when graphed as points on a coordinate plane? Justify your reasoning.

_____

_____

_____

9. **Persevere with Problems** The constant rate of change for the relationship shown in the table is $8 per hour. Find the missing values.

Time (h)	1	2	3
Earnings ($)	x	y	z

$x =$ _____ $y =$ _____ $z =$ _____

Name _____ My Homework _____

Extra Practice

Find the constant rate of change for each table.

10.

Time (h)	0	1	2	3
Wage ($)	0	9	18	27

$\frac{\text{change in wages}}{\text{change in hours}} = \frac{\$9}{1 \text{ hour}}$ $9 per hour

Homework Help

11.

Minutes	1,000	1,500	2,000	2,500
Cost ($)	38	53	68	83

_____

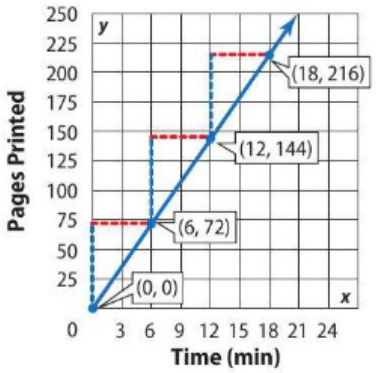
12. Use the graph to find the constant rate of change. Then, explain what the points (0, 0) and (6, 72) represent.

_____

_____

_____

_____



13. CCSS **Justify Conclusions** Ramona and Josh earn money by babysitting. The amounts earned for one evening are shown in the table and graph. Who charged more per hour? Explain.

Ramona's Earnings	
Time (hours)	Earnings ($)
2	18
3	27
4	36

_____

_____

14. The cost of 1 movie ticket is $7.50. The cost of 2 movie tickets is $15. Based on this constant rate of change, what is the cost of 4 movie tickets? _____

Think Smart for Smarter Balanced

15. Reggie started a running program to prepare for track season. He ran a half hour each morning for 60 days. He averaged 6.5 miles per hour. What is the total number of miles Reggie ran over the 60-day period? ☐

16. Select the correct constant rate of change for each table of data.

Number of Apples	3	7	11
Number of Seeds	30	70	110

☐ $\frac{1}{12}$ ☐ $\frac{1}{10}$ ☐ $\frac{12}{1}$ ☐ $\frac{10}{1}$

Number of Tables	4	6	9
Number of Chairs	48	72	108

☐ $\frac{1}{12}$ ☐ $\frac{1}{10}$ ☐ $\frac{12}{1}$ ☐ $\frac{10}{1}$

Number of Passengers	24	60	120
Number of Vans	2	5	10

☐ $\frac{1}{12}$ ☐ $\frac{1}{10}$ ☐ $\frac{12}{1}$ ☐ $\frac{10}{1}$

Number of Booklets	20	50	100
Number of Pages	2	5	10

☐ $\frac{1}{12}$ ☐ $\frac{1}{10}$ ☐ $\frac{12}{1}$ ☐ $\frac{10}{1}$

Common Core Spiral Review

Write the output for each given input in the tables below. 5.OA.3

17.

Input	Add 4	Output
1	1 + 4	
2	2 + 4	
3	3 + 4	
4	4 + 4	

18.

Input	Subtract 5	Output
30	30 − 5	
40	40 − 5	
50	50 − 5	
60	60 − 5	

19.

Input	Multiply by 2	Output
1	1 × 2	
2	2 × 2	
3	3 × 2	
4	4 × 2	

20.

Input	Divide by 3	Output
3	3 ÷ 3	
6	6 ÷ 3	
9	9 ÷ 3	
12	12 ÷ 3	

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Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions eHelp

Find the constant rate of change for each table. (Example 1)

1.

Time (s)	Distance (m)
1	6
2	12
3	18
4	24

6 m per s

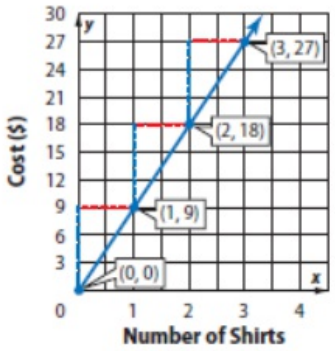
2.

Items	Cost ($)
2	18
4	36
6	54
8	72

$9 per item

3. The graph shows the cost of purchasing T-shirts. Find the constant rate of change for the graph. Then explain what points (0, 0) and (1, 9) represent. (Examples 2 and 3)

$9 per shirt; Sample answer: The point (0, 0) represents 0 T-shirts purchased and 0 dollars spent. The point (1, 9) represents 9 dollars spent for 1 T-shirt.



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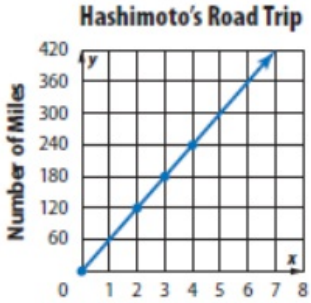

the Guzman family; Sample answer: The unit rate for the Guzmans is 45 miles per hour. The unit rate for Hashimotos is 60 miles per hour.

5. At 1:00 P.M., the water level in a pool is 13 inches. At 1:30 P.M., the water level is 18 inches. At 2:30 P.M., the water level is 28 inches. What is the constant rate of change?

10 inches per hour

PG 69 1-5

Name _____ My Homework _____

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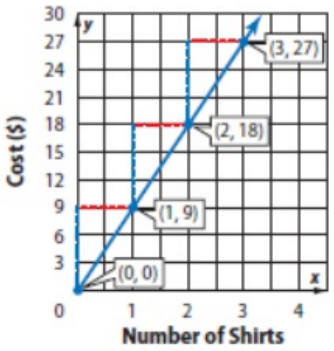

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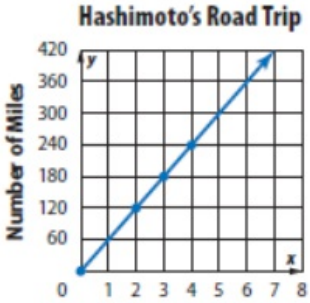

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10 inches per hour

PG 69 1-5

Name _____ My Homework _____

Independent Practice

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Find the constant rate of change for each table. (Example 1)

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3	18
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6 m per s

2.

Items	Cost ($)
2	18
4	36
6	54
8	72

$9 per item

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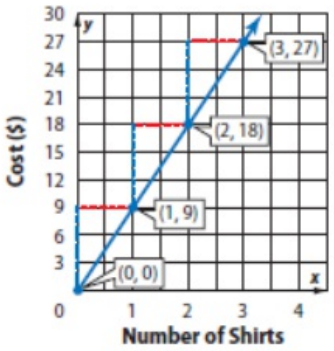

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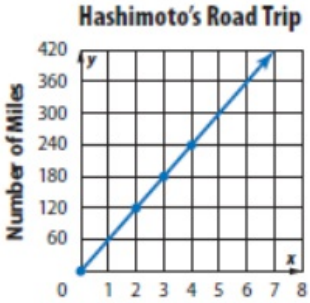

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5. At 1:00 P.M., the water level in a pool is 13 inches. At 1:30 P.M., the water level is 18 inches. At 2:30 P.M., the water level is 28 inches. What is the constant rate of change?

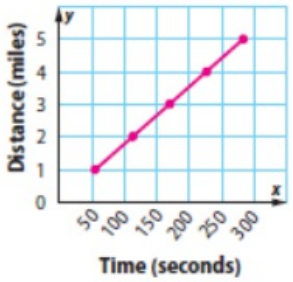
10 inches per hour

PG 70, Ø

6 **Model with Mathematics** Refer to the lap times for Exercises **a** and **b**.

a. How long does it take Seth to race 1 mile? Write the constant rate of change in miles per second. Round to the nearest hundredth. 57.1 s; about 0.02 mi per s

b. Graph the ordered pairs (time, distance) on the coordinate plane at the right. Connect the points with a solid line.



H.O.T. Problems

7. **Model with Mathematics** Make a table where the constant rate of change is 6 inches for every foot.

Sample answer:

Feet	Inches
3	18
6	36
9	54
12	72

8. **Justify Conclusions** The terms in sequence A increase by 3. The terms in sequence B increase by 8. In which sequence do the terms form a steeper line when graphed as points on a coordinate plane? Justify your reasoning.

Sample answer: sequence B; Since the common difference is greater, its terms increase at a faster rate and the points form a steeper line.

9. **Persevere with Problems** The constant rate of change for the relationship shown in the table is $8 per hour. Find the missing values.

Time (h)	1	2	3
Earnings ($)	x	y	z

$x =$ 8 $y =$ 16 $z =$ 24

PG 71, 13

Name _____ My Homework _____

Extra Practice

Find the constant rate of change for each table.

10.

Time (h)	0	1	2	3
Wage ($)	0	9	18	27

$\frac{\text{change in wages}}{\text{change in hours}} = \frac{\$9}{1 \text{ hour}}$ $9 per hour

Homework Help

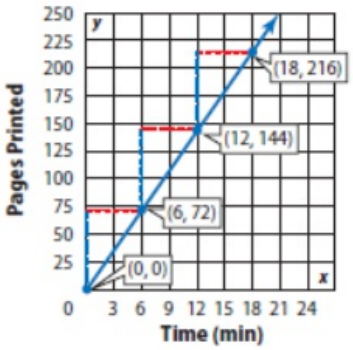
11.

Minutes	1,000	1,500	2,000	2,500
Cost ($)	38	53	68	83

$0.03 per minute

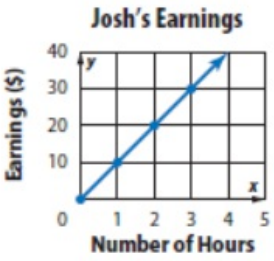
12. Use the graph to find the constant rate of change. Then, explain what the points (0, 0) and (6, 72) represent.

12 pages per minute; Sample answer: The point (0, 0) represents zero pages printed in zero minutes. The point (6, 72) represents printing 72 pages in 6 minutes.



13. CCSS **Justify Conclusions** Ramona and Josh earn money by babysitting. The amounts earned for one evening are shown in the table and graph. Who charged more per hour? Explain.

Ramona's Earnings	
Time (hours)	Earnings ($)
2	18
3	27
4	36



Josh; sample answer: The unit rate for Ramona is $9 per hour. The unit rate for Josh is $10 per hour.

14. The cost of 1 movie ticket is $7.50. The cost of 2 movie tickets is $15. Based on this constant rate of change, what is the cost of 4 movie tickets? $30

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16-20

Think Smart for Smarter Balanced

15. Reggie started a running program to prepare for track season. He ran a half hour each morning for 60 days. He averaged 6.5 miles per hour. What is the total number of miles Reggie ran over the 60-day period? **195 mi**

16. Select the correct constant rate of change for each table of data.

Number of Apples	3	7	11
Number of Seeds	30	70	110

☐ $\frac{1}{12}$ ☐ $\frac{1}{10}$ ☐ $\frac{12}{1}$ ■ $\frac{10}{1}$

Number of Tables	4	6	9
Number of Chairs	48	72	108

☐ $\frac{1}{12}$ ☐ $\frac{1}{10}$ ■ $\frac{12}{1}$ ☐ $\frac{10}{1}$

Number of Passengers	24	60	120
Number of Vans	2	5	10

■ $\frac{1}{12}$ ☐ $\frac{1}{10}$ ☐ $\frac{12}{1}$ ☐ $\frac{10}{1}$

Number of Booklets	20	50	100
Number of Pages	2	5	10

☐ $\frac{1}{12}$ ■ $\frac{1}{10}$ ☐ $\frac{12}{1}$ ☐ $\frac{10}{1}$

CCSS Common Core Spiral Review

Write the output for each given input in the tables below. 5.OA.3

17.

Input	Add 4	Output
1	1 + 4	5
2	2 + 4	6
3	3 + 4	7
4	4 + 4	8

18.

Input	Subtract 5	Output
30	30 − 5	25
40	40 − 5	35
50	50 − 5	45
60	60 − 5	55

19.

Input	Multiply by 2	Output
1	1 × 2	2
2	2 × 2	4
3	3 × 2	6
4	4 × 2	8

20.

Input	Divide by 3	Output
3	3 ÷ 3	1
6	6 ÷ 3	2
9	9 ÷ 3	3
12	12 ÷ 3	4