PAGE 17

Lesson 2

Complex Fractions and Unit Rates

What You'll Learn

List two headings you would use to make an outline of the lesson.

- _____
- _____

Essential Question

HOW can you show that two objects are proportional?

Vocabulary

complex fraction

Common Core State Standards

Content Standards
7.RP.1, 7.NS.3

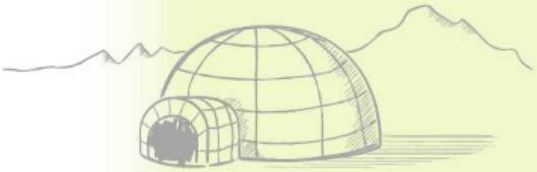
Mathematical Practices
1, 3, 4, 6

Real-World Link

Speed Skating Dana is skating laps to train for a speed skating competition. She can skate 1 lap in 40 seconds.

1. Write a ratio in simplest form comparing Dana's time to her number of laps.

SPEED = $\frac{\text{Dana's time (s)} \rightarrow 40}{\text{Number of Laps} \rightarrow 1}$



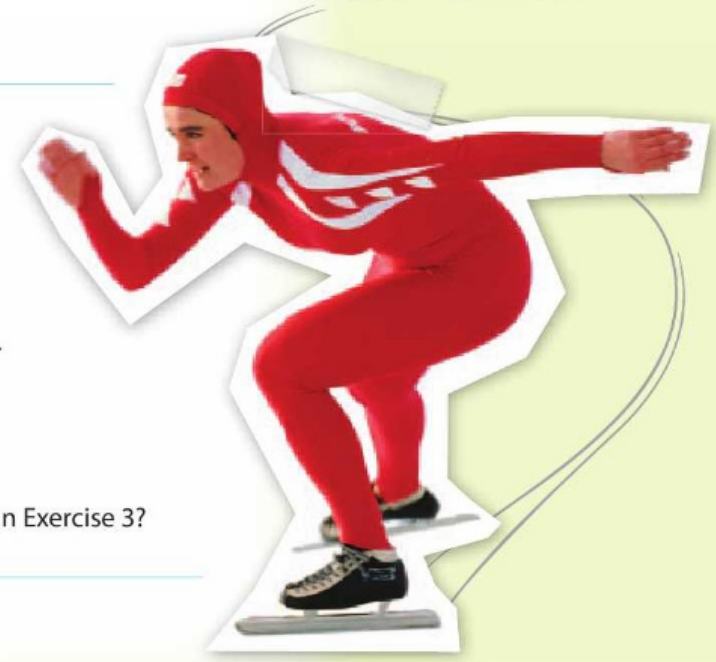
2. Suppose Dana skates for 20 seconds. How many laps will she skate?

$0.5 = \frac{1}{2}$

3. Write the ratio of Dana's time from Exercise 2 to her number of laps.

$\frac{\text{Dana's time} \rightarrow 20}{\text{Number of Laps} \rightarrow 2}$ $\quad 1$

4. How could you simplify the ratio you wrote in Exercise 3?



Work Zone

Simplify a Complex Fraction

Fractions like $\frac{20}{\frac{1}{2}}$ are called complex fractions. **Complex fractions** are fractions with a numerator, denominator, or both that are also fractions. Complex fractions are simplified when both the numerator and denominator are integers.

Examples

Tutor

1. Simplify $\frac{\frac{1}{4}}{2}$.

FRACTION LINE SHOWS DIVISION

Recall that a fraction can also be written as a division problem.

$\frac{\frac{1}{4}}{2} = \frac{1}{4} \div 2$ Write the complex fraction as a division problem.

$= \frac{1}{4} \times \frac{1}{2}$ Multiply by the reciprocal of 2, which is $\frac{1}{2}$.

$= \frac{1}{8}$ Simplify. THE RECIPROCAL OF $\frac{2}{3}$ IS $\frac{3}{2}$

So, $\frac{\frac{1}{4}}{2}$ is equal to $\frac{1}{8}$.

Divide Fractions

To divide by a whole number, first write it as a fraction with a denominator of 1. Then multiply by the reciprocal.

So, $\frac{\frac{1}{4}}{2}$ can be written as $\frac{1}{4} \div \frac{2}{1}$.

RECIPROCAL

$\frac{2}{3} \times \boxed{\frac{3}{2}} = \frac{6}{6} = 1$

$\frac{1}{5} \times \boxed{\frac{5}{1}} = \frac{5}{5} = 1$

2. Simplify $\frac{1}{\frac{1}{2}}$.

$-\frac{6}{11} \times \boxed{-\frac{11}{6}} = \frac{66}{66} = 1$

Write the fraction as a division problem.

$\frac{1}{\frac{1}{2}} = 1 \div \frac{1}{2}$ Write the complex fraction as a division problem.

$= \frac{1}{1} \times \frac{2}{1}$ Multiply by the reciprocal of $\frac{1}{2}$, which is $\frac{2}{1}$.

$= \frac{2}{1}$ or 2 Simplify.

So, $\frac{1}{\frac{1}{2}}$ is equal to 2.

ORIGINAL NUMBER	RECIPROCAL
$-\frac{2}{3}$	$-\frac{3}{2}$
$\frac{1}{5}$	$\frac{5}{1} = 5$
$\frac{7}{10}$	$\frac{10}{7}$
$\frac{4}{3}$	$\frac{3}{4}$
$7 = \frac{7}{1}$	$\frac{1}{7}$

Show your work.

a. _____

b. _____

c. _____

d. _____

Got It? Do these problems to find out.

a. $\frac{2}{\frac{2}{3}}$ (DIVISION; 2 NUMERATOR, $\frac{2}{3}$ DENOMINATOR)

b. $\frac{6}{\frac{1}{3}}$

c. $\frac{\frac{2}{3}}{7} = \frac{2}{21}$

$\frac{2}{3} \div \frac{7}{1}$

$\frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$

d. $\frac{\frac{2}{4}}{2}$

a. $2 \div \frac{2}{3}$

$\frac{2}{1} \times \frac{3}{2} = \frac{2 \times 3}{1 \times 2} = \frac{6}{2} = 3$

Find Unit Rates

When the fractions of a complex fractions represent different units, you can find the unit rate.

Examples

Tutor

3. **Josiah can jog $1\frac{1}{3}$ miles in $\frac{1}{4}$ hour. Find his average speed in miles per hour.**

Write a rate that compares the number of miles to hours.

$$\frac{1\frac{1}{3}\text{ mi}}{\frac{1}{4}\text{ h}} = 1\frac{1}{3} \div \frac{1}{4}$$ Write the complex fraction as a division problem.

$$= \frac{4}{3} \div \frac{1}{4}$$ Write the mixed number as an improper fraction.

$$= \frac{4}{3} \times \frac{4}{1}$$ Multiply by the reciprocal of $\frac{1}{4}$, which is $\frac{4}{1}$.

$$= \frac{16}{3} \text{ or } 5\frac{1}{3}$$ Simplify.

So, Josiah jogs at an average speed of $5\frac{1}{3}$ miles per hour.

4. **Tia is painting her house. She paints $34\frac{1}{2}$ square feet in $\frac{3}{4}$ hour. At this rate, how many square feet can she paint each hour?**

Write a ratio that compares the number of square feet to hours.

$$\frac{34\frac{1}{2}\text{ ft}^2}{\frac{3}{4}\text{ h}} = 34\frac{1}{2} \div \frac{3}{4}$$ Write the complex fraction as a division problem.

$$= \frac{69}{2} \div \frac{3}{4}$$ Write the mixed number as an improper fraction.

$$= \frac{69}{2} \times \frac{4}{3}$$ Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{276}{6} \text{ or } 46$$ Simplify.

So, Tia can paint 46 square feet per hour.

Got It? Do these problems to find out.

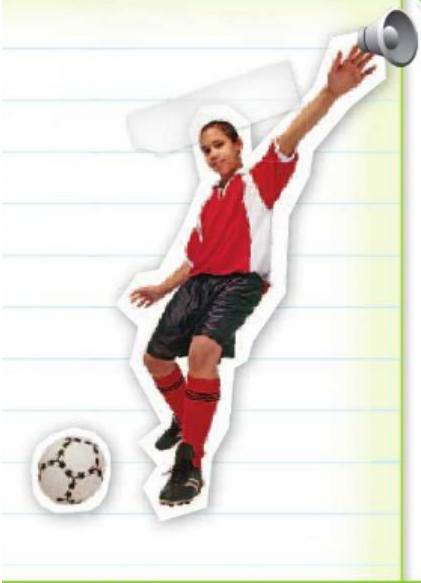
Show your work.

e. Mr. Ito is spreading mulch in his yard. He spreads $4\frac{2}{3}$ square yards in 2 hours. How many square yards can he mulch per hour?

e. _____

f. Aubrey can walk $4\frac{1}{2}$ miles in $1\frac{1}{2}$ hours. Find her average speed in miles per hour.

f. _____



Example

5. **On Javier's soccer team, about $33\frac{1}{3}\%$ of the players have scored a goal. Write $33\frac{1}{3}\%$ as a fraction in simplest form.**

$$33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100}$$ Definition of percent

$$= 33\tfrac{1}{3} \div 100$$ Write the complex fraction as a division problem.

$$= \frac{100}{3} \div 100$$ Write $33\frac{1}{3}$ as an improper fraction.

$$= \frac{\overset{1}{\cancel{100}}}{3} \times \frac{1}{\underset{1}{\cancel{100}}}$$ Multiply by the reciprocal of 100, which is $\frac{1}{100}$.

$$= \frac{1}{3}$$ Simplify.

So, about $\frac{1}{3}$ of Javier's team has scored a goal.

Guided Practice

Simplify. (Examples 1 and 2)

1. $\dfrac{18}{\frac{3}{4}} =$ _____

2. $\dfrac{\frac{3}{6}}{4} =$ _____

3. $\dfrac{\frac{1}{3}}{\frac{1}{4}} =$ _____

Show your work.

4. Pep Club members are making spirit buttons. They make 490 spirit buttons in $3\frac{1}{2}$ hours. Find the number of buttons the Pep Club makes per hour. (Examples 3 and 4) _____

5. A county sales tax is $6\frac{2}{3}\%$. Write the percent as a fraction in simplest form. (Example 5) _____

6. **Building on the Essential Question** What is a complex fraction? _____

Rate Yourself!

How confident are you about simplifying complex fractions? Check the box that applies.

For more help, go online to access a Personal Tutor.

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions eHelp

Simplify. (Examples 1 and 2)

1. $\frac{1}{\frac{2}{3}} =$ _____

2. $\frac{2}{\frac{3}{11}} =$ _____

3. $\frac{\frac{8}{9}}{6} =$ _____

4. $\frac{\frac{2}{5}}{9} =$ _____

5. $\frac{\frac{4}{5}}{10} =$ _____

6. $\frac{\frac{1}{4}}{\frac{7}{10}} =$ _____

7. Mary is making pillows for her Life Skills class. She bought $2\frac{1}{2}$ yards of fabric. Her total cost was $15. What was the cost per yard? (Examples 3 and 4)

_____

8. Doug entered a canoe race. He rowed $3\frac{1}{2}$ miles in $\frac{1}{2}$ hour. What is his average speed in miles per hour? (Examples 3 and 4)

_____

9. Monica reads $7\frac{1}{2}$ pages of a mystery book in 9 minutes. What is her average reading rate in pages per minute? (Examples 3 and 4) _____

Write each percent as a fraction in simplest form. (Example 5)

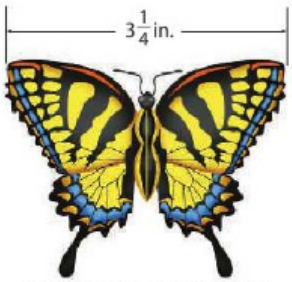
10. $56\frac{1}{4}\% =$ _____

11. $15\frac{3}{5}\% =$ _____

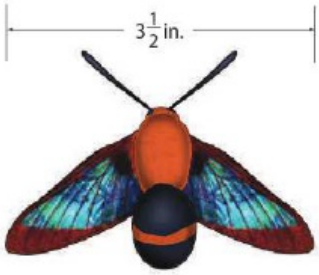
12. $13\frac{1}{3}\% =$ _____

13. A bank is offering home loans at an interest rate of $5\frac{1}{2}\%$. Write the percent as a fraction in simplest form. (Example 5) _____

14. **Be Precise** Karl measured the wingspan of the butterfly and the moth shown below. How many times larger is the moth than the butterfly?



Black Swallowtail Butterfly



Hummingbird Moth

H.O.T. Problems Higher Order Thinking

15. **Construct an Argument** Explain how complex fractions can be used to solve problems involving ratios.

16. **Reason Inductively** Write three different complex fractions that simplify to $\frac{1}{4}$.

17. **Persevere with Problems** Use mental math to find the value of $\frac{15}{124} \cdot \frac{230}{30} \div \frac{230}{124}$.

18. **Justify Conclusions** The value of a mutual fund increased by $3\frac{1}{8}\%$. Write $3\frac{1}{8}\%$ as a fraction in simplest form. Justify your answer.

19. **Persevere with Problems** The distance around the tire of a motorized scooter is 21.98 inches. The tires make one revolution every $\frac{1}{10}$ second. Find the speed of the scooter in miles per hour. Round to the nearest tenth. (*Hint: The speed of an object spinning in a circle is equal to the distance around the circle divided by the time it takes to complete one revolution.*)

Name _____ My Homework _____

Extra Practice

Simplify.

20. $\frac{1}{\frac{1}{4}} =$ 4

$\frac{1}{\frac{1}{4}} = 1 \div \frac{1}{4}$

$= \frac{1}{1} \times \frac{4}{1}$

$= \frac{4}{1}$ or 4

Homework Help

21. $\frac{12}{\frac{3}{5}} =$ _____

22. $\frac{\frac{9}{10}}{9} =$ _____

23. $\frac{\frac{1}{2}}{\frac{1}{4}} =$ _____

24. $\frac{\frac{1}{12}}{\frac{5}{6}} =$ _____

25. $\frac{\frac{5}{6}}{\frac{5}{9}} =$ _____

26. Mrs. Frasier is making costumes for the school play. Each costume requires 0.75 yard of fabric. She bought 6 yards of fabric. How many costumes can Mrs. Frasier make?

_____

27. A lawn company advertises that they can spread 7,500 square feet of grass seed in $2\frac{1}{2}$ hours. Find the number of square feet of grass seed that can be spread per hour.

_____

Write each percent as a fraction in simplest form.

28. $2\frac{2}{5}\% =$ _____

29. $7\frac{3}{4}\% =$ _____

30. $8\frac{1}{3}\% =$ _____

31. CCSS **Justify Conclusions** The value of a certain stock increased by $1\frac{1}{4}\%$. Explain how to write $1\frac{1}{4}\%$ as a fraction in simplest form. _____

_____

_____

Think Smart for Smarter Balanced

32. Debra bought $3\frac{1}{4}$ yards of fabric at a remnant sale for $13. Determine if each of the following remnant deals have the same unit price as Debra's deal. Select yes or no.

a. $4\frac{2}{3}$ yards for $16 ☐ Yes ☐ No

b. $2\frac{3}{4}$ yards for $11 ☐ Yes ☐ No

c. $6\frac{1}{2}$ yards for $26 ☐ Yes ☐ No

33. The table shows the distances traveled by 4 cyclists. Sort the speeds of the riders, in miles per hour, from slowest to fastest.

	Rider	Speed (mph)
Slowest		
Fastest		

Bicycle Rides

Rider	Distance	Time
Elena	$20\frac{1}{2}$ mi	$2\frac{1}{4}$ h
Julio	$12\frac{1}{4}$ mi	$1\frac{1}{2}$ h
Kevin	$20\frac{2}{3}$ mi	$1\frac{2}{3}$ h
Lorena	$33\frac{1}{4}$ mi	$2\frac{1}{3}$ h

Which rider had the fastest rate of speed? ☐

CCSS Common Core Spiral Review

Fill in each box with the equivalent customary measurement. 5.MD.1

34. 2 feet = ☐ inches

35. 5 tons = ☐ pounds

36. 8 gallons = ☐ quarts

Fill in each box with the equivalent metric measurement. 5.MD.1

37. 1 meter = ☐ centimeters

38. 1 liter = ☐ milliliters

39. 1 kilogram = ☐ grams