

Proportional and Nonproportional Relationships

What You'll Learn

Scan the lesson. Write the definitions of proportional and nonproportional.

• ~~proportional~~

PROPORTION → TWO EQUIVALENT RATIOS

• ~~nonproportional~~

$$\frac{\text{FLOUR}}{\text{TOTAL}} = \frac{2}{3} = \frac{8}{12}$$

$$2 \cdot 12 = 24$$

$$3 \cdot 8 = 24$$



Essential Question

HOW can you show that two objects are proportional?



Vocabulary

proportional
nonproportional
equivalent ratios



Common Core State Standards

Content Standards
7.RP.2, 7.RP.2a, 7.RP.2b

Mathematical Practices
1, 3, 4

Real-World Link

Pizza Party Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges \$8 per medium pizza.

- Complete the table to determine the cost for different numbers of pizzas ordered.

Cost (\$)	8	16	24	32	40	80
Pizza	1	2	3	4	5	10

- For each number of pizzas, fill in the boxes to write the relationship of the cost and number of pizzas as a ratio in simplest form.

UNIT RATE

$$\frac{16}{2} = \frac{8}{1} \quad \frac{24}{3} = \frac{8}{1}$$

$$\frac{32}{4} = \frac{8}{1} \quad \frac{40}{5} = \frac{8}{1}$$

- What do you notice about the simplified ratios?



Identify Proportional Relationships

Two quantities are **proportional** if they have a constant ratio or unit rate. For relationships in which this ratio is not constant, the two quantities are **nonproportional**.

In the pizza example on the previous page, the cost of an order is *proportional* to the number of pizzas ordered.

$$\frac{\text{cost of order}}{\text{pizzas ordered}} = \frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4} = \frac{40}{5} \text{ or } \$8 \text{ per pizza}$$

All of the ratios above are **equivalent ratios** because they all have the same value.



Example



- 1. Andrew earns \$18 per hour for mowing lawns. Is the amount of money he earns proportional to the number of hours he spends mowing? Explain.**

Find the amount of money he earns for working a different number of hours. Make a table to show these amounts.

Earnings (\$)	18	36	54	72
Time (h)	1	2	3	4

For each number of hours worked, write the relationship of the amount he earned and hour as a ratio in simplest form.

$$\frac{\text{amount earned}}{\text{number of hours}} \rightarrow \frac{18}{1} \text{ or } \frac{18}{1} \quad \frac{36}{2} \text{ or } \frac{18}{1} \quad \frac{54}{3} \text{ or } \frac{18}{1} \quad \frac{72}{4} \text{ or } \frac{18}{1}$$

All of the ratios between the two quantities can be simplified to 18.

The amount of money he earns is proportional to the number of hours he spends mowing.

Show your work.

Got It? Do this problem to find out.

- a. At Lakeview Middle School, there are 2 homeroom teachers assigned to every 48 students. Is the number of students at this school proportional to the number of teachers? Explain your reasoning.

STUDENTS	48	96	144
TEACHERS	2	4	6

$$\frac{48}{2} = \frac{24}{1}$$

$$\frac{96}{4} = \frac{24}{1}$$

$$\frac{144}{6} = \frac{24}{1}$$



Examples



2. Uptown Tickets charges \$7 per baseball game ticket plus a \$3 processing fee per order. Is the cost of an order proportional to the number of tickets ordered? Explain.

$$\frac{10}{1} \neq \frac{17}{2}$$

Cost (\$)	10	17	24	31
Tickets Ordered	1	2	3	4

For each number of tickets, write the relationship of the cost and number of tickets as a ratio in simplest form.

$$\frac{\text{cost of order}}{\text{tickets ordered}} \rightarrow \frac{10}{1} \text{ or } 10 \quad \frac{17}{2} \text{ or } 8.5 \quad \frac{24}{3} \text{ or } 8 \quad \frac{31}{4} \text{ or } 7.75$$

Since the ratios of the two quantities are not the same, the cost of an order is *not* proportional to the number of tickets ordered.



3. You can use the recipe shown to make a fruit punch. Is the amount of sugar used proportional to the amount of mix used? Explain.

Find the amount of sugar and mix needed for different numbers of batches. Make a table to help you solve.

Cups of Sugar	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
Envelopes of Mix	1	2	3	4

For each number of cups of sugar, write the relationship of the cups and number of envelopes of mix as a ratio in simplest form.

$$\frac{\text{cups of sugar}}{\text{envelopes of mix}} \rightarrow \frac{\frac{1}{2}}{1} \text{ or } 0.5 \quad \frac{1}{2} \text{ or } 0.5 \quad \frac{1\frac{1}{2}}{3} \text{ or } 0.5 \quad \frac{2}{4} \text{ or } 0.5$$

All of the ratios between the two quantities can be simplified to 0.5. The amount of mix used is proportional to the amount of sugar used.



Got It? Do this problem to find out.

- b. At the beginning of the year, Isabel had \$120 in the bank. Each week, she deposits another \$20. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

Time (wk)	1	2	3
Balance (\$)	120	140	160

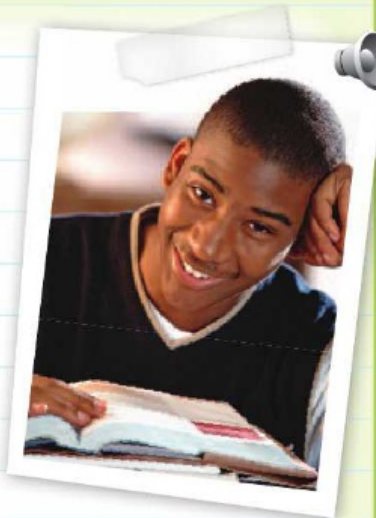
NON PROPORTIONAL RELATIONSHIP

$$\frac{120}{1} = \text{SIMPLIFIED}$$

$$\frac{140}{2} = \frac{70}{1}$$

Show your work.

b. _____



Example



4. The tables shown represent the number of pages Martin and Gabriel read over time. Which situation represents a proportional relationship between the time spent reading and the number of pages read? Explain.

Pages Martin Read	2	4	6
Time (min)	5	10	15

Pages Gabriel Read	3	4	7
Time (min)	5	10	15

Write the ratios for each time period in simplest form.

$\frac{\text{pages}}{\text{minutes}} \rightarrow \frac{2}{5}, \frac{4}{10} \text{ or } \frac{2}{5}, \frac{6}{15} \text{ or } \frac{2}{5} \quad \frac{3}{5}, \frac{4}{10} \text{ or } \frac{2}{5}, \frac{7}{15}$

All of the ratios between Martin's quantities are $\frac{2}{5}$. So, Martin's reading rate represents a proportional relationship.

Guided Practice



1. The Vista Marina rents boats for \$25 per hour. In addition to the rental fee, there is a \$12 charge for fuel. Use a table to determine if the number of hours you rent the boat is proportional to the total cost. Explain. (Examples 1–3)

Rental Time (h)			
Cost (\$)			



2. Which situation represents a proportional relationship between the hours worked and amount earned for Matt and Jane? Explain. (Example 4)

Matt's Earnings (\$)	12	20	31
Time (h)	1	2	3

Jane's Earnings (\$)	12	24	36
Time (h)	1	2	3

3.  **Building on the Essential Question** Explain what makes two quantities proportional.

Rate Yourself!

How confident are you about determining proportional relationships? Shade the ring on the target.



For more help, go online to access a Personal Tutor.



FOLDABLES Time to update your Foldable!

Independent Practice

Go online for Step-by-Step Solutions



For Exercises 1 and 2, use a table to solve. Then explain your reasoning.

(Examples 1 and 2)

- 1** An adult elephant drinks about 225 liters of water each day. Is the number of days the water supply lasts proportional to the number of liters of water the elephant drinks?

Time (days)	1	2	3	4
Water (L)	225	450		

- 2.** An elevator *ascends*, or goes up, at a rate of 750 feet per minute. Is the height to which the elevator ascends proportional to the number of minutes it takes to get there? (Examples 1–3)

Time (min)	1	2	3	4
Height (ft)				

- 3.** Which situation represents a proportional relationship between the number of laps run by each student and their time? (Example 4)

Desmond's Time (s)	146	292	584
Laps	2	4	8

Maria's Time (s)	150	320	580
Laps	2	4	6

Copy and Solve Use a table to help you solve. Then explain your reasoning. Show your work on a separate piece of paper.

- 4.** Plant A is 18 inches tall after one week, 36 inches tall after two weeks, 56 inches tall after three weeks. Plant B is 18 inches tall after one week, 36 inches tall after two weeks, 54 inches tall after three weeks. Which situation represents a proportional relationship between the plants' height and number of weeks? (Example 4)

- 5** Determine whether the measures for the figure shown are proportional.
- the length of a side and the perimeter
 - the length of a side and the area

