Lesson 3-2 Rational Numbers

Interactive Study Guide

See pages 53–54 for:
- Getting Started
- Vocabulary Start-Up
- Notes

Essential Question

What happens when you add, subtract, multiply, and divide rational numbers?

Common Core State Standards

Content Standards
7.NS.2, 7.NS.2d, 8.NS.1, 7.EE.3

Mathematical Practices
1, 3, 4, 7

Vocabulary

rational numbers

What You'll Learn
- Write rational numbers as fractions.
- Identify and classify rational numbers.

Real-World Link

Monkeys New World monkeys are primates that are found in Mexico, Central America, and South America. Their common characteristics include being small to mid-sized, possessing high intelligence, and being skilled in using their hands. In a similar way, numbers can also be organized into sets based on shared characteristics.

Rational Numbers

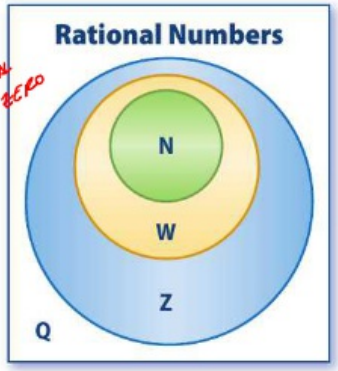
When you first learned to count using the numbers 1, 2, 3, …, you were using members of the set of *natural numbers*, N.

If you add zero to the set of natural numbers, the result is the set of *whole numbers*, W = {0, 1, 2, 3, …}.

Whole numbers and their opposites make up the set of *integers*, Z = {…, −3, −2, −1, 0, 1, 2, 3, …}.

Any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ is part of the set of **rational numbers**, Q. Some examples of rational numbers are shown below.

$$0.87 \quad -23 \quad \frac{2}{3} \quad -2.\overline{56} \quad 1\frac{1}{2}$$



Example 1

Write each rational number as a fraction.

a. $6\frac{1}{6}$

$6\frac{1}{6} = \frac{37}{6}$ Write $6\frac{1}{6}$ as an improper fraction.

b. -23

$-23 = \frac{-23}{1}$ or $-\frac{23}{1}$

Got It? Do these problems to find out.

1a $4\frac{2}{3} = \frac{14}{3}$

1b. $7 = \frac{7}{1}$

$4\frac{2}{3} \rightarrow 4 \times 3 + 2 = \frac{14}{3}$

$1 = \frac{3}{3}$

$\frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3} =$

$\frac{3+3+3+3+2}{3} = \frac{14}{3}$

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Terminating and repeating decimals are rational numbers because they can be written as fractions

Decimal Point
Use the word *and* to represent the decimal point.
- Read 0.625 as *six hundred twenty-five thousandths.*
- Read 20.005 as *twenty and five thousandths.*

Example 2

a. Write 0.64 as a fraction in simplest form.

$0.64 = \frac{64}{100}$ 0.64 is 64 hundredths.

$= \frac{16}{25}$ The GCF of 64 and 100 is 4.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten-thousandths
0	0	0	0	6	4	0	0

b. A handheld video game system weighs 9.675 ounces. Write this decimal as a mixed number in simplest form.

$9.675 = 9\frac{675}{1000}$ 0.675 is 675 thousandths.

$= 9\frac{27}{40}$ The GCF of 675 and 1000 is 25.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten-thousandths
0	0	0	9	6	7	5	0

$0.84 = \frac{84}{100} \div \frac{4}{4} = \frac{21}{25}$ $21 = 3 \cdot 7$, $25 = 5 \cdot 5$

Got It? Do these problems to find out.

Write each decimal as a fraction in simplest form. $5\frac{875}{1000} \div \frac{25}{25} = 5\frac{35}{40} \div \frac{5}{5} = 5\frac{7}{8}$

2a. 0.84

2b. 5.875

2c. Rock music accounted for 0.35 of the total music sales in a recent year. Write this decimal as a fraction in simplest form. $\frac{35}{100} \div \frac{5}{5} = \frac{7}{20}$

Repeating Decimals
When *two* digits repeat, multiply each side by 100. Then subtract N from $100N$ to eliminate the repeating part.

Example 3

Write $0.\overline{6}$ as a fraction in simplest form.

$0.\overline{6}$

$N = 0.6666\ldots$ Let N represent the number.

$10N = 10(0.6666\ldots)$ Multiply each side by 10 because one digit repeats.

$10N = 6.666\ldots$

Subtract N from $10N$ to eliminate the repeating part, 0.666.... $N = 0.\overline{42}$

COEFFICIENT

$10N = 6.666\ldots$

$-N = 0.666\ldots$

$9N = 6$ $10N - N = 10N - 1N$ or $9N$

$\frac{9N}{9} = \frac{6}{9}$ Divide each side by 9.

$N = \frac{6}{9}$ or $\frac{2}{3}$

0.424...

0.4242...

0.42424...

Check 6 ÷ 9 ENTER 0.666666667 ✓

Got It? Do this problem to find out.

3. Write $0.\overline{42}$ as a fraction in simplest form.

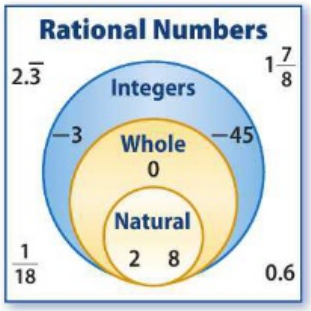
Ratios

Rational comes from the word ratio. A *ratio* is the comparison of two quantities by division. Recall that $\frac{a}{b} = a \div b$, where $b \neq 0$.

Key Concept Rational Numbers

Watch

A rational number is any number that can be expressed as the quotient $\frac{a}{b}$, where a and b are integers and $b \neq 0$.



All rational numbers can be written as terminating or repeating decimals. Decimals that neither terminate nor repeat, such as the numbers below, are called *irrational numbers*. You will learn more about irrational numbers in Chapter 4.

$\pi = 3.141592\ldots$ ⋯➤ The digits do not repeat.

$8.787787778\ldots$ ⋯➤ The same block of digits does not repeat.

Example 4

Tutor

Identify all sets to which each number belongs.

a. $-2\frac{6}{11}$

Since $-2\frac{6}{11}$ can be written as $-\frac{28}{11}$, it is rational.

b. 1.313313331...

This is a nonterminating and nonrepeating decimal. So, it is irrational.

c. 45

45 is a natural number, a whole number, an integer, and a rational number.

Got It? Do these problems to find out.

4a. 0 **4b.** $1\frac{4}{5}$ **4c.** 1.414213562...

Guided Practice

Check

Write each number as a fraction. (Example 1)

1. $3\frac{3}{4}$ **2.** -9 **3.** $-1\frac{3}{4}$

Write each decimal as a fraction or mixed number in simplest form. (Examples 2 and 3)

4. 0.07 **5.** $-3.\overline{85}$ **6.** $0.\overline{78}$

7. There are approximately 2.54 centimeters in 1 inch. Express 2.54 as a mixed number.

Identify all sets to which each number belongs. (Example 4)

8. −632 **9.** $0.\overline{56}$ **10.** 21

Independent Practice

Go online for Step-by-Step Solutions — eHelp

Write each number as a fraction. (Example 1)

11. $1\frac{5}{6}$ **12.** -12 **13.** $-10\frac{7}{8}$ **14.** 49

Write each decimal as a fraction or mixed number in simplest form. (Example 2)

15. 3.625 **16.** 0.55 **17.** −5.36

18. −0.265 **19.** −1.3 **20.** 0.9

21. Financial Literacy Recently, one U.S. dollar was equal to 0.506 British pounds. Express 0.506 as a fraction in simplest form. (Example 2)

22. The estimated portions for various age groups of the population for 2010 are shown in the table. (Example 2)

Age Group	Portion of Population
19 years and under	0.27
20 to 64 years	0.60
65 years and over	0.13

a. Find the fraction in simplest form of the population that is 19 years of age or younger.

b. Find the fraction in simplest form of the population that is 20 to 64 years of age.

Write each decimal as a fraction or mixed number in simplest form. (Example 3)

23. $-2.\overline{5}$ **24.** $0.\overline{36}$ **25.** 0.161616…

26. $9.\overline{27}$ **27.** $-0.\overline{09}$ **28.** $-10.\overline{74}$

Identify all sets to which each number belongs. (Example 4)

29. −8 **30.** 14 **31.** 9.23

32. $1\frac{5}{9}$ **33.** 0.323322333… **34.** 3.141516…

35. Maria has a bead that is 0.6 inch long. She wants to use the bead to fill a space that is $\frac{5}{8}$ inch long. Will the bead fit? Explain.

36. All of the Calories in one cup of milk come from fat, protein, and carbohydrates. Use the table to find the fraction of Calories that comes from protein. Write the fraction in simplest form.

Nutrient	Decimal Part of Calories
fat	0.03
protein	■
carbohydrates	0.53

Replace each ● with <, >, or = to make a true sentence.

37. −0.23 ● −0.3 **38.** $\frac{8}{9}$ ● 0.888…

39. 0.714 ● $\frac{5}{7}$ **40.** $-1\frac{1}{11}$ ● −0.9

41. $4.\overline{63}$ ● $4\frac{5}{8}$ **42.** $-5.\overline{3}$ ● 5.333…

Write each decimal as a fraction or mixed number in simplest form.

43. $0.\overline{652}$ **44.** $0.1\overline{8}$

45. $0.72\overline{4}$ **46.** $3.5\overline{96}$

47. $9.2\overline{43}$ **48.** $0.24\overline{67}$

b. $3.14 < 3.1415927 < 3.142857$

49. **Multiple Representations** Pi (π) is a nonrepeating, nonterminating decimal. Two common estimates for pi are 3.14 and $\frac{22}{7}$.

a. Graph Use a calculator to find the value of π to seven decimal places. Graph this value, 3.14, and $\frac{22}{7}$ on a number line.

b. Symbols Write an inequality comparing the values.

c. Words To find the circumference of a circle, you multiply pi by the diameter d of the circle. Explain when you might use 3.14 to find the circumference and when you might use $\frac{22}{7}$ to find the circumference.

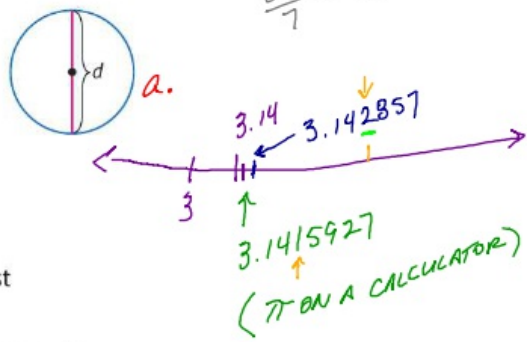
3.1415927

$\frac{22}{7} = 3.142857$

c. $C = \pi d$

$d = 14$

$\frac{22}{7}\left(\frac{14}{1}\right) = 44$



50. The mathematician Archimedes believed that π was between $3\frac{1}{7}$ and $3\frac{10}{71}$.

a. Express each mixed number as a decimal rounded to the nearest thousandth. Was Archimedes' theory correct? Explain.

b. The Rhind Papyrus records that the Egyptians used $\frac{256}{81}$ for π. Express the fraction as a decimal rounded to the nearest thousandth. Which value is closer to the actual value of π, Archimedes' or the Egyptians' value?

Order each set of rational numbers from least to greatest.

51. $-3.4, 3\frac{4}{11}, -3.\overline{42}, 3.38$

52. $\frac{1}{3}, 0.\overline{13}, \frac{5}{13}, 0.32$

53. $-1\frac{13}{14}, -1.9, -1\frac{9}{11}, -1.95$

54. $9\frac{4}{5}, 9.\overline{79}, 9\frac{11}{13}, 9.82$

55. A lion's speed is $\frac{5}{7}$ the speed of a cheetah. Find the least rational number with a denominator of 9 that is greater than $\frac{5}{7}$. Find the greatest rational number with a denominator of 8 that is less than $\frac{5}{7}$. Write an inequality comparing the three numbers.

H.O.T. Problems Higher Order Thinking

56. **Model with Mathmatics** Choose a repeating decimal in which three digits repeat. Write the number as a fraction or mixed number in simplest form.

57. **Construct an Argument** Explain why $0.\overline{76}$ is greater than 0.76.

58. **Persevere with Problems** Antonio stated that $0.\overline{9} = 1$. Show that he is correct.

59. **Use a Counterexample** Determine whether the following statements are *true* or *false*. If true, explain your reasoning. If false, give a counterexample.

a. All integers are rational numbers.

b. All whole numbers are integers.

c. A rational number is always an integer.

d. All natural numbers are rational.

60. **Building on the Essential Question** How do you compare and order fractions and decimals? Give an example to explain your reasoning.

Standardized Test Practice

61. Which fraction is between 0.12 and 0.15?

A $\frac{3}{25}$ **C** $\frac{3}{20}$

B $\frac{1}{8}$ **D** $\frac{1}{5}$

62. Which of the following is not a rational number?

F $\frac{4}{9}$ **H** $0.\overline{62}$

G -4.27 **J** $-3.131131113\ldots$

63. Last football season, Jason made 0.85 of his field goal attempts. Write this decimal as a fraction in simplest form.

A $\frac{85}{100}$ **C** $\frac{17}{20}$

B $\frac{20}{17}$ **D** $\frac{100}{85}$

64. Short Response The table shows the results of a survey about how students get to school.

Method of Transportation	Portion of Students
bus	0.40
walk	0.18
car	0.36
bicycle	0.04
other	0.02

Which method of transportation do most students use to get to school? Write the fraction of students who use this method of transportation in simplest form.

Common Core Review

Write each fraction as a decimal. Use a bar to show a repeating decimal. 8.NS.1

65. $-\frac{5}{8}$ **66.** $\frac{1}{6}$ **67.** $-\frac{2}{10}$ **68.** $\frac{4}{7}$

69. Ms. Adepoju grades students' exams by starting with a perfect score of 100, and marking points off for incorrect answers. The table shows the results of 5 students on a recent exam. 6.NS.7

a. Write an integer to describe each student's grade with respect to a perfect score of 100.

b. Order the students from highest to lowest grades.

Student	Points Taken Off
Ava	5
Brennan	3
Denny	6
Jose	0
Hao	7

Convert each measurement to the given units. 5.MD.1

70. 24 feet to inches **71.** 24 ounces to pounds **72.** 300 minutes to hours

73. Mount Kilimanjaro's elevation is 5895 meters. Lake Assal's elevation is −155 meters. Find the difference between these elevations. 7.NS.1

Find each product. 7.NS.2, 7.NS.2a

74. $-6(-12)$ **75.** $15(-3)(-4)(0)$ **76.** $-3(5)(-9)$ **77.** $14(-20)$

Find each quotient. 7.NS.2, 7.NS.2b

78. $\frac{1}{-1}$ **79.** $\frac{-16}{8}$ **80.** $\frac{-100}{-10}$ **81.** $\frac{0}{-5}$

$\sqrt{1} = 1$ $\sqrt{2} = 1.$

$\sqrt{4} = 2$ $\sqrt{3} = 1.$