



## Lesson 4-2

## Negative Exponents

**Interactive Study Guide**

See pages 75–76 for:

- Getting Started
- Real-World Link
- Notes

**Essential Question**

Why is it useful to write numbers in different ways?

**Common Core State Standards**

Content Standards  
8.EE.1

Mathematical Practices  
1, 3, 4, 6

**Vocabulary**

negative exponent

Handwritten notes showing the pattern of powers of 2:

$$\begin{aligned}
 2^3 &= 8 \xrightarrow{\div 2} 4 \\
 2^2 &= 4 \xrightarrow{\div 2} 2 \\
 2^1 &= 2 \xrightarrow{\div 2} 1 \\
 2^0 &= 1 \\
 2^{-1} &= 0.5 = \frac{1}{2} \\
 2^{-2} &= 0.25 = \frac{1}{4} \\
 2^{-3} &= 0.125 = \frac{1}{8}
 \end{aligned}$$

**What You'll Learn**

- Write expressions using negative exponents.
- Evaluate numerical expressions containing negative exponents.

**Real-World Link**

**Snowflakes** Have you ever heard that no two snowflakes are exactly alike? This is because they are made up of water molecules, which grow at varying patterns and rates depending on humidity, air currents, and time in the air. Negative exponents are useful for describing small measures, like the diameter of snowflakes.


**Key Concept Negative and Zero Exponents**

**Symbols** For  $a \neq 0$  and any whole number  $n$ ,  $a^{-n} = \frac{1}{a^n}$ .  
For  $a \neq 0$ ,  $a^0 = 1$ .

**Example**  $8^{-2} = \frac{1}{8^2}$      $x^0 = 1, x \neq 0$

A **negative exponent** is the result of repeated division. Extending the pattern below shows that  $\frac{1}{100}$  or  $\frac{1}{10^2}$  can be defined as  $10^{-2}$ .



Exponential Form	Standard Form
$10^3 = 10 \cdot 10 \cdot 10$	1000
$10^2 = 10 \cdot 10$	100
$10^1$	10
$10^0$	1
$10^{-1}$	$\frac{1}{10}$
$10^{-2}$	$\frac{1}{100}$
$10^{-3}$	$\frac{1}{1000}$

$\div 10$   
 $\div 10$   
 $\div 10$   
 $\div 10$   
 $\div 10$

Handwritten notes showing the pattern of powers of 3:

$$\begin{aligned}
 3^3 &= 27 \\
 3^2 &= 9 \\
 3^1 &= 3 \\
 3^0 &= 1 \\
 3^{-1} &= \frac{1}{3} = \frac{1}{3^1} \\
 3^{-2} &= \frac{1}{9} = \frac{1}{3^2} \\
 3^{-3} &= \frac{1}{27} = \frac{1}{3^3}
 \end{aligned}$$

**Watch Out!**

Remember  $2^{-3}$  equals  $\frac{1}{2^3}$ , not  $-8$ .

**Example 1**

Write each expression using a positive exponent.

a.  $2^{-3}$

$$2^{-3} = \frac{1}{2^3}$$

Definition of negative exponent

b.  $m^{-4}$

$$m^{-4} = \frac{1}{m^4}$$

Definition of negative exponent

$$1 \div 2 = 1 \times \frac{1}{2}$$

**Got It?** Do these problems to find out.

1a.  $3^{-5} = \frac{1}{3^5}$

1b.  $y^{-3} = \frac{1}{y^3}$

1c.  $2^0 = 1$

$$\frac{1}{3^5} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$$

**Division by Zero**

Since division by zero is undefined, assume that no denominator equals zero. So, in Example 1b,  $m \neq 0$ .

**Example 2**

Write each fraction as an expression using a negative exponent other than  $-1$ .

a.  $\frac{1}{4^2}$

$$\frac{1}{4^2} = 4^{-2}$$

Definition of negative exponent

b.  $\frac{1}{100}$

$$\frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

Definition of exponent

Definition of negative exponent

**Got It?** Do these problems to find out.

2a.  $\frac{1}{6^3} = 6^{-3}$

2b.  $\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

2c.  $\frac{1}{27} = \frac{1}{3^3} = 3^{-3}$



Negative exponents are often used in science when dealing with very small numbers. Usually the number is a power of 10.

**Example 3**

**STEM** Geckos have tiny hairs on the bottom of their feet that are about 0.000001 meter long. Write the decimal as a fraction and as a power of 10.

$$0.000001 = \frac{1}{1,000,000}$$

Write the decimal as a fraction.

$$= \frac{1}{10^6}$$

$$1,000,000 = 10^6$$

$$= 10^{-6}$$

Definition of negative exponent

4 ← EXPONENT  
2 ← BASE

Therefore, 0.000001 is  $\frac{1}{1,000,000}$  as a fraction and  $10^{-6}$  as a power of 10.

**Got It?** Do these problems to find out.

3a. The slowest-moving fish is a sea horse. It swims at a maximum speed of 0.0001 mile per minute. Write the decimal as a fraction and as a power of ten.

3b. The smallest species of ant has a mass of 0.00001 gram. Write the decimal as a fraction and as a power of ten.

$$\begin{aligned} 1000 &= 10^3 \\ 100 &= 10^2 \\ 10 &= 10^1 \\ 1 &= 10^0 \\ \frac{1}{10} &= 10^{-1} \\ \frac{1}{100} &= 10^{-2} \end{aligned}$$

$$\frac{1}{10000} = 10^{-4}, \frac{1}{10^4}$$



**Evaluate Expressions**

Algebraic expressions containing negative exponents can be written using positive exponents and then evaluated.

**Order of Operations**

Remember to follow the order of operations when evaluating expressions.

**Example 4**

Evaluate  $4a^{-5}$  if  $a = -2$ .

$$\begin{aligned}
 4a^{-5} &= 4 \cdot (-2)^{-5} && \text{Replace } a \text{ with } -2. \\
 &= 4 \cdot \frac{1}{(-2)^5} && \text{Definition of negative exponent} \\
 &= 4 \cdot \frac{1}{-32} && \text{Find } (-2)^5. \\
 &= \cancel{4}^1 \cdot \frac{1}{\cancel{32}^{-8}} && \text{Simplify.} \\
 &= \frac{1}{-8} && \text{Simplify.}
 \end{aligned}$$

**Got It?** Do these problems to find out.

Evaluate each expression if  $m = 4$  and  $n = 3$ .

4a.  $m^{-2}$       4b.  $6mn^{-4}$       4c.  $-n^{-3}$       4d.  $-4m^{-2}$

**Guided Practice**

Write each expression using a positive exponent. (Example 1)

1.  $6^{-2}$       2.  $(-2)^{-3}$       3.  $x^{-5}$       4.  $b^{-7}$

Write each fraction as an expression using a negative exponent other than  $-1$ . (Example 2)

5.  $\frac{1}{2^6}$       6.  $\frac{1}{8^2}$       7.  $\frac{1}{9}$       8.  $\frac{1}{36}$



9. When a baseball is hit, it comes in contact with the bat for less than 0.001 of a second. Write 0.001 using a negative exponent other than  $-1$ . (Example 3)

Evaluate each expression if  $x = -4$  and  $y = 2$ . (Example 4)

10.  $y^{-7}$       11.  $x^{-3}$       12.  $3^x$       13.  $8y^{-4}$

**Independent Practice**

Go online for Step-by-Step Solutions



Write each expression using a positive exponent. (Example 1)

14.  $11^{-6}$       15.  $7^{-1}$       16.  $(-4)^{-5}$       17.  $(-5)^{-4}$   
 18.  $a^{-2}$       19.  $k^{-8}$       20.  $b^{-15}$       21.  $r^{-20}$

Write each fraction as an expression using a negative exponent other than  $-1$ . (Example 2)

22.  $\frac{1}{9^4}$       23.  $\frac{1}{10^3}$       24.  $\frac{1}{7^6}$       25.  $\frac{1}{6^5}$   
 26.  $\frac{1}{4}$       27.  $\frac{1}{49}$       28.  $\frac{1}{144}$       29.  $\frac{1}{125}$

$$3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

$$3^4 = 81$$

$$3^{-1} = \frac{1}{3^1} = \frac{1}{3} = 1 \div 3$$

$$\frac{1}{2} = 0.5$$

$$\rightarrow 3^{-4} = \frac{1}{3^4} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{81}$$

$$2 \overline{) 1.0} \\ \underline{1.0} \\ 0$$

$$\frac{1}{3^4} = \left(\frac{1}{3}\right)^4 = \frac{1^4}{3} = \frac{1}{3}$$

$$3^{-4} = \frac{1}{3^4}$$

$$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

$$5^4 = 5 \cdot 5 \cdot 5 \cdot 5 = 625$$

$$(-5)^4 = -5 \cdot (-5) \cdot (-5) \cdot (-5) = 625$$

$$(-5)^{-4} = \frac{1}{-5} \cdot \frac{1}{-5} \cdot \frac{1}{-5} \cdot \frac{1}{-5} = \frac{1}{625}$$

$\downarrow$   
 $-\frac{1}{5}$