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Terminating and Repeating Decimals

What You'll Learn

Scan the lesson. List two headings you would use to make an outline of the lesson.

- $\frac{3}{4} = 0.75$ ← TERMINATING DECIMAL IT ENDS
- $\frac{2}{3} = 0.666666\ldots = 0.\overline{6}$ ← BAR NOTATION ← REPEATING DECIMAL

Essential Question

WHAT happens when you add, subtract, multiply, and divide fractions?

Vocabulary

repeating decimal
bar notation
terminating decimal

Common Core State Standards

Content Standards
7.NS.2, 7.NS.2d, 7.EE.3

Mathematical Practices
1, 3, 4, 6, 7

Vocabulary Start-Up

Any fraction can be expressed as a decimal by dividing the numerator by the denominator.

The decimal form of a fraction is called a **repeating decimal**. Repeating decimals can be represented using **bar notation**. In bar notation, a bar is drawn only over the digit(s) that repeat.

$0.3333\ldots = 0.\overline{3}$ $0.1212\ldots = 0.\overline{12}$ $11.38585\ldots = 11.3\overline{85}$

If the repeating digit is zero, the decimal is a **terminating decimal**. The terminating decimal $0.25\overline{0}$ is typically written as 0.25.

Match each repeating decimal to the correct bar notation.

$0.1111\ldots$	$0.6\overline{1}$
$0.61111\ldots$	$0.\overline{1}$
$0.616161\ldots$	$0.\overline{61}$

Real-World Link

Jamie had two hits on her first nine times at bat. To find her batting "average," she divided 2 by 9.

$2 \div 9 = 0.2222\ldots$

Write 0.2222… using bar notation. $\frac{4}{9} = 0.\overline{4}$

Round 0.2222… to the nearest thousandth. $\frac{8}{9} = 0.\overline{8}$

$\frac{2}{9} = 0.\overline{2}$

$\frac{2}{3} = \frac{6}{9} = 0.\overline{6}$

Work Zone

Write Fractions as Decimals

Our decimal system is based on powers of 10 such as 10, 100, and 1,000. If the denominator of a fraction is a power of 10, you can use place value to write the fraction as a decimal.

Complete the table below. Write fractions in simplest form.

Words	Fraction	Decimal
seven tenths	$\frac{7}{10}$	0.7
nineteen hundredths	$\frac{19}{100}$	0.19
one-hundred five thousandths	$\frac{105}{1000}$	0.105

If the denominator of a fraction is a *factor* of 10, 100, 1,000, or any greater power of ten, you can use mental math and place value.

Examples

Tutor

Write each fraction or mixed number as a decimal.

1. $\frac{74}{100}$

Use place value to write the equivalent decimal.

$\frac{74}{100} = 0.74$ Read $\frac{74}{100}$ as *seventy-four hundredths.*

So, $\frac{74}{100} = 0.74$.

2. $\frac{7}{20}$

Think $\frac{7}{20} = \frac{35}{100}$ (× 5 / × 5)

So, $\frac{7}{20} = 0.35$.

3. $5\frac{3}{4}$

$5\frac{3}{4} = 5 + \frac{3}{4}$ Think of it as a sum.

$= 5 + 0.75$ You know that $\frac{3}{4} = 0.75$.

$= 5.75$ Add mentally.

So, $5\frac{3}{4} = 5.75$.

Show your work.

a. _____

b. _____

c. _____

Got It? Do these problems to find out.

a. $\frac{3}{10} = 0.3$ b. $\frac{3}{25} = 0.12$ c. $-6\frac{1}{2} = -6.5$

$\frac{3}{25} \times \frac{4}{4} = \frac{12}{100}$

$\frac{1}{2} = 0.5$

Examples

Tutor

4. Write $\frac{3}{8}$ as a decimal.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -24 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Divide 3 by 8.

Division ends when the remainder is 0.

So, $\frac{3}{8} = 0.375$.

5. Write $-\frac{1}{40}$ as a decimal.

$$\begin{array}{r} 0.025 \\ 40\overline{)1.000} \\ -80 \\ \hline 200 \\ -200 \\ \hline 0 \end{array}$$

Divide 1 by 40.

So, $-\frac{1}{40} = -0.025$.

6. Write $\frac{7}{9}$ as a decimal.

$$\begin{array}{r} 0.777\ldots \\ 9\overline{)7.000} \\ -63 \\ \hline 70 \\ -63 \\ \hline 70 \\ -63 \\ \hline 7 \end{array}$$

Divide 7 by 9.

Notice that the division will never terminate in zero.

So, $\frac{7}{9} = 0.777\ldots$ or $0.\overline{7}$.

Got It? Do these problems to find out.

Write each fraction or mixed number as a decimal. Use bar notation if needed.

d. $-\frac{7}{8}$ = $-7 \div 8$

e. $2\frac{1}{8}$

f. $-\frac{3}{11}$

g. $8\frac{1}{3}$

$-\frac{7}{8} = -0.875$

$$\begin{array}{r} 0.875 \\ 8\overline{)7.0} \\ 64 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

$\frac{7}{8} < 1$

$8\frac{1}{3} = 8.\overline{3}$

$$\begin{array}{r} 0.33 \\ 3\overline{)10} \\ 9 \\ \hline 10 \\ 9 \\ \hline 10 \end{array}$$

Bar Notation

Remember that you can use bar notation to indicate a number pattern that repeats indefinitely.

$0.333\ldots = 0.\overline{3}$.

Show your work.

d. _____

e. _____

f. _____

g. _____

Write Decimals as Fractions

Every terminating decimal can be written as a fraction with a denominator of 10, 100, 1,000, or a greater power of ten. Use the place value of the final digit as the denominator.

STOP and Reflect

Suppose 0.6 of the fish are goldfish. Write this decimal as a fraction in the space below.

Example

Watch Tutor

7. **Find the fraction of the fish in the aquarium that are goldfish. Write in simplest form.**

Fish	Amount
Angelfish	0.4
Goldfish	0.15
Guppy	0.25
Molly	0.2

$0.15 = \frac{15}{100}$ The digit 5 is in the hundredths place.

$= \frac{3}{20}$ Simplify.

So, $\frac{3}{20}$ of the fish are goldfish.

Show your work.

h. _____

i. _____

j. _____

Got It? **Do these problems to find out.**

Determine the fraction of the aquarium made up by each fish. Write the answer in simplest form.

h. molly **i.** guppy **j.** angelfish

Guided Practice

Check

Write each fraction or mixed number as a decimal. Use bar notation if needed. (Examples 1–6)

Show your work.

1. $\frac{2}{5} =$ _____

2. $-\frac{9}{10} =$ _____

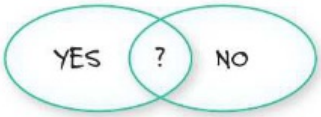
3. $\frac{5}{9} =$ _____

4. During a hockey game, an ice resurfacer travels 0.75 mile. What fraction represents this distance? (Example 7)

5. **Building on the Essential Question** How can you write a fraction as a decimal?

Rate Yourself!

Are you ready to move on? Shade the section that applies.



For more help, go online to access a Personal Tutor.

Tutor

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions eHelp

Write each fraction or mixed number as a decimal. Use bar notation if needed. (Examples 1–6)

1. $\frac{1}{2} =$ _____

Show your work.

2. $-4\frac{4}{25} =$ -4.16

$\frac{4}{25} \cdot \frac{4}{4} = \frac{16}{100}$

3. $\frac{1}{8} =$ _____

4. $\frac{3}{16} =$ _____

$\frac{3}{9}$

$0.\overline{3}$

5. $-\frac{33}{50} =$ -0.66

$\frac{33}{50} \cdot \frac{2}{2} = \frac{66}{100}$

6. $-\frac{17}{40} =$ _____

7. $5\frac{7}{8} =$ _____

8. $9\frac{3}{8} =$ _____

9. $-\frac{8}{9} =$ $-0.\overline{8}$

0.88
$9\overline{)8.0}$
72
80
72
80

10. $-\frac{1}{6} =$ _____

11. $-\frac{8}{11} =$ _____

12. $2\frac{6}{11} =$ _____

Write each decimal as a fraction or mixed number in simplest form. (Example 7)

13. $-0.2 =$ $-\frac{1}{5}$

$\frac{2}{10} = \frac{1}{5}$

14. $0.55 =$ _____

15. $5.96 =$ $5\frac{24}{25}$

0.96

$\frac{96}{100} = \frac{48}{50} = \frac{24}{25}$

16. The screen on Brianna's new phone is 2.85 centimeters long. What mixed number represents the length of the phone screen? (Example 7)

_____

17. STEM A praying mantis is an interesting insect that can rotate its head 180 degrees. Suppose the praying mantis at the right is 10.5 centimeters long. What mixed number represents this length? (Example 7)

$10.5 = 10\frac{1}{2}$

$0.5 = \frac{1}{2}$

18. **Persevere with Problems** Suppose you buy a 1.25-pound package of ham at $5.20 per pound.

a. What fraction of a pound did you buy?

b. How much money did you spend?

H.O.T. Problems Higher Order Thinking

19. **Identify Structure** Write a fraction that is equivalent to a terminating decimal between 0.5 and 0.75.

20. **Persevere with Problems** Fractions in simplest form that have denominators of 2, 4, 8, 16, and 32 produce terminating decimals. Fractions with denominators of 6, 12, 18, and 24 produce repeating decimals. What causes the difference? Explain.

21. **Persevere with Problems** The value of pi (π) is 3.1415926… . The mathematician Archimedes believed that π was between $3\frac{1}{7}$ and $3\frac{10}{71}$. Was Archimedes correct? Explain your reasoning.

22. **Reason Inductively** A *unit fraction* is a fraction that has 1 as its numerator. Write the four greatest unit fractions that are repeating decimals. Then write each fraction as a decimal.

23. **Model with Mathematics** Write a real-world scenario in which it would be appropriate to write a value in fractional form.

Name _____ My Homework _____

Extra Practice

Write each fraction or mixed number as a decimal. Use bar notation if needed.

24. $\frac{4}{5} =$ 0.8

Homework Help

$\frac{4}{5} = \frac{8}{10}$ (×2)

So, $\frac{4}{5} = 0.8$.

25. $-7\frac{1}{20} =$ _____

26. $-\frac{4}{9} =$ _____

27. $5\frac{1}{3} =$ _____

28. The fraction of a dime that is made up of copper is $\frac{12}{16}$. Write this fraction as a decimal.

_____

Write each decimal as a fraction or mixed number in simplest form.

29. $-0.9 =$ _____

30. $0.34 =$ _____

31. $2.66 =$ _____

Write each of the following as an improper fraction.

32. $-13 =$ _____

33. $7\frac{1}{3} =$ _____

34. $-3.2 =$ _____

35. CCSS **Be Precise** Nicolás practiced playing the cello for 2 hours and 18 minutes. Write the time Nicolás spent practicing as a decimal.

_____

Think Smart for Smarter Balanced

36. The table shows the lengths of four hiking trails. Select the appropriate decimal equivalent of each trail length.

1.2	1.25	1.3	$1.\overline{3}$
1.6	$1.\overline{6}$	1.75	

Hiking Trail	Trail Length	Decimal Equivalents
Lakeview	$1\frac{1}{4}$	1.25
Forest Lane	$1\frac{1}{3}$	$1.\overline{3}$
Sparrow Stroll	$1\frac{3}{10}$	1.3
Mountain Climb	$1\frac{2}{3}$	$1.\overline{6}$

$\frac{1}{3} = 0.33333\ldots = 0.\overline{3}$

37. Zoe went to lunch with a friend. After tax, her bill was $12.05. Which of the following rational numbers is equivalent to this amount? Select all that apply.

- ☐ $12\frac{1}{20}$
- ☐ $\frac{25}{2}$
- ☐ $\frac{241}{20}$
- ☐ $12\frac{5}{100}$

CCSS Common Core Spiral Review

Round each decimal to the tenths place. 5.NBT.4

38. 5.69 ≈ _____

39. 0.05 ≈ _____

40. 98.99 ≈ _____

Graph and label each fraction on the number line below. 6.NS.6

41. $\frac{1}{2}$

42. $\frac{3}{4}$

43. $\frac{2}{3}$

44. The table shows the discount on athletic shoes at two stores selling sporting equipment. Which store is offering the greater discount? Explain. 6.NS.7

Store	Discount
Good Sports	$\frac{1}{5}$
Go Time	25%

_____

_____