Lesson 7-5

Factoring Linear Expressions

Interactive Study Guide

See pages 159–160 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

Why are algebraic rules useful?

Common Core State Standards

Content Standards
7.EE.1

Mathematical Practices
1, 2, 3, 4, 7

Vocabulary

factor

factored form

What You'll Learn

- Find the greatest common factor of two monomials.
- Use properties to factor linear expressions.

Real-World Link

Marching Band Band directors create geometrical formations that are eye-catching and exciting but still follow the rhythm and feel of the music. Graph paper is used to draw formations, with different colored ink representing different sections of the band.

Find the GCF of Monomials

To **factor** a number means to write it as a product of its factors. A monomial can be factored using the same method you would use to factor a number. The greatest common factor (GCF) of two monomials is the greatest monomial that is a factor of both.

Example 1

Tutor

Find the GCF of each pair of monomials.

a. $4x, 12x$

$4x = 2 \cdot 2 \cdot x$

$12x = 2 \cdot 2 \cdot 3 \cdot x$

Write the prime factorization of $4x$ and $12x$.

Circle the common factors.

The GCF of $4x$ and $12x$ is $2 \cdot 2 \cdot x$ or $4x$.

b. $18a, 20ab$

$18a = 2 \cdot 3 \cdot 3 \cdot a$

$20ab = 2 \cdot 2 \cdot 5 \cdot a \cdot b$

Write the prime factorization of $18a$ and $20ab$.

Circle the common factors.

The GCF of $18a$ and $20ab$ is $2 \cdot a$ or $2a$.

Got It? Do these problems to find out.

Find the GCF of each pair of monomials.

1a. $12, 28c$ **1b.** $25x, 15xy$ **1c.** $42mn, 14mn$

Factor Linear Expressions

You can use the Distributive Property and the work backward strategy to express an algebraic expression as a product of its factors. An algebraic expression is in **factored form** when it is expressed as the product of its factors.

$8x + 4 = 4(2x) + 4(1)$ The GCF of $8x$ and 4 is 4.

$= 4(2x + 1)$ Distributive Property

Example 2

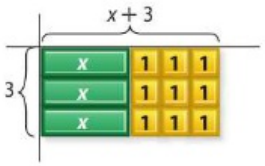
Tutor

Factor each expression.

a. $3x + 9$

Method 1 Use a model.

Model $3x + 9$.



Arrange the tiles into equal rows and columns. The rectangle has a width of three 1-tiles, or 3, and a length of one x-tile and three 1-tiles, or $x + 3$.

So, $3x + 9 = 3(x + 3)$.

Method 2 Use the GCF.

$3x = 3 \cdot x$ Write the prime factorization of $3x$ and 9.

$9 = 3 \cdot 3$ Circle the common factors.

The GCF of $3x$ and 9 is 3. Write each term as a product of the GCF and its remaining factors.

$3x + 9 = \mathbf{3}(x) + \mathbf{3}(3)$

$= \mathbf{3}(x + 3)$ Distributive Property

So, $3x + 9 = 3(x + 3)$.

Factoring Expressions

Use algebra tiles to model the expression in Example 2b. Since you cannot rearrange the tiles to make a rectangle, the expression cannot be factored.

b. $12x + 7$

Find the GCF of $12x$ and 7.

$12x = 2 \cdot 2 \cdot 3 \cdot x$

$7 = 1 \cdot 7$

There are no common factors, so $12x + 7$ *cannot be factored.*

2a. $4x + 28$

$4(x + 7)$

2c. $4x + 35$ CANNOT BE FACTORED

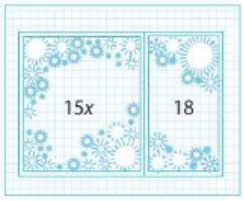
Got It? Do these problems to find out.

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed.

2a. $4x + 28$ **2b.** $3 + 33x$ **2c.** $4x + 35$

Example 3

The garden at the right has a total area of $(15x + 18)$ square feet. Find possible dimensions of the garden.



Factor $15x + 18$.

$15x = 3 \cdot 5 \cdot x$ Write the prime factorization of $15x$ and 18.

$18 = 2 \cdot 3 \cdot 3$ Circle the common factors.

The GCF of $15x$ and 18 is 3. Write each term as a product of the GCF and its remaining factors.

$15x + 18 = 3(5x) + 3(6)$

$= 3(5x + 6)$ Distributive Property

So, the dimensions of the garden are 3 feet and $(5x + 6)$ feet.

Check Find the product of 3 and $5x + 6$. $3(5x + 6) = 15x + 18$ ✓

Got It? Do these problems to find out.

3a. Financial Literacy The Reyes family has saved \$480 as a down payment for a new television. If x is the monthly payment for one year, the expression $\$12x + \$480$ represents the total cost of the television. Factor $\$12x + \$480$.

3b. Jesse wants to put down \$100 toward a new computer and will pay it off in six months. If y is the monthly payment, what expression represents the total price?

Guided Practice

Find the GCF of each pair of monomials. (Example 1)

1. $32x, 18$ **2.** $15y, 25$ **3.** $45a, 20a$

4. $16b, 12b$ **5.** $42s, 28s$ **6.** $56g, 84gh$

7. $27s, 54st$ **8.** $18cd, 30cd$ **9.** $22mn, 11kmn$

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed. (Example 2)

10. $36x + 24$ **11.** $6 + 3x$ **12.** $4x + 9$

13. $13x + 21$ **14.** $2x - 4$ **15.** $14x - 16$

16. $12 + 18x$ **17.** $24 + 32x$ **18.** $15x + 8$

19. Mr. Phen's monthly income can be represented by the expression $25x + 120$, where x is the number of hours worked. Factor the expression $25x + 120$. (Example 3)

20. The area of a high school basketball court is $(50x - 300)$ square feet. Factor $50x - 300$ to find possible dimensions of the basketball court. (Example 3)

Independent Practice

Go online for Step-by-Step Solutions

Find the GCF of each pair of monomials. (Example 1)

21. 24, 48*m*

22. 63*p*, 84

23. 40*x*, 60*x*

24. 32*a*, 48*b*

25. 30*rs*, 42*rs*

26. 54*gh*, 72*g*

27. 36*k*, 144*km*

28. 60*jk*, 45*jkm*

29. 100*xy*, 75*xyz*

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed. (Example 2)

30. $3x + 9$

31 $5x + 5$

32. $10x - 35$

33. $2x - 15$

34. $4x - 7$

35. $32 + 24x$

36. $12 + 30x$

37. $18x + 6$

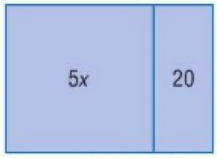
38. $30x - 40$

39. The area of a rectangle is $(4x - 8)$ square units. Factor $4x - 8$ to find possible dimensions of the rectangle. (Example 3)

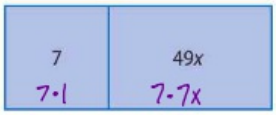
40. James has \$120 in his savings account and plans to save \$$x$ each month for 6 months. The expression $\$6x + \$120$ represents the total amount in the account after 6 months. Factor the expression $6x + 120$. (Example 3)

Write an expression in factored form to represent the total area of each rectangle.

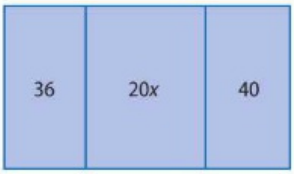
41.



42.



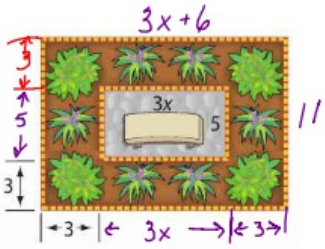
43.



44. A square scrapbooking page has a perimeter of $(8x + 20)$ inches. What is the length of one side of the page?

45 Six friends visited a museum to see the new holograms exhibit. The group paid for admission to the museum and \$12 for parking. The total cost of the visit can be represented by the expression $\$6x + \$12$. What was the cost of the visit for one person?

46. CCSS **Reason Abstractly** The diagram represents a flower border that is 3 feet wide surrounding a rectangular sitting area. Write an expression in factored form that represents the area of the flower border.



Write an expression in factored form that is equivalent to the given expression.

47. $\frac{1}{2}x + 4$

48. $\frac{2}{3}x + 6$

49. $\frac{3}{4}x - 24$

50. $\frac{5}{6}x - 30$

39.

$$4\overline{)4x+8} \quad x+2$$

$$4(x+2)$$

$$4\overline{)32} \quad 8$$

$$4(8)$$

$$2\overline{)4x+8} \quad 2x+4$$

$$2(2x+4)$$

$$1\overline{)4x+8} \quad 4x+8$$

$$1(4x+8)$$

$$\tfrac{1}{2}\overline{)4x+8} \quad 8x+16$$

$$\tfrac{1}{2}(8x+16)$$

$$2\overline{)4x-8} \quad 2x-4$$

$$2(2x-4)$$

$$4\overline{)4x-8} \quad x-2$$

$$4(x-2)$$

$x > 2$

$\frac{4}{10} = \frac{2 \cdot 2}{5 \cdot 2} = \frac{2}{5}$

GCF

$\frac{4x}{10xy} =$

$\frac{2 \cdot 2 \cdot x}{2 \cdot 5 \cdot x \cdot y} = \frac{2}{5y}$

GCF $2x$

PROBLEM 39

$2x - 4$

2 | $A = 4x - 8$

$8x - 16$

$\frac{1}{2}$ | $4x - 8$

$x - 2$

4 | $A = 4x - 8$

H.O.T. Problems Higher Order Thinking

51. **Identify Structure** Write two monomials whose greatest common factor is $4m$.

52. **Find the Error** Enrique is factoring $90x - 15$. Find his mistake and correct it.

$90x - 15 = 15(6x)$
$= 90x$

53. **Building on the Essential Question** Explain how the GCF is used to factor an expression. Use the term *Distributive Property* in your response.

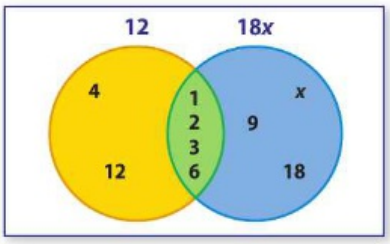
Standardized Test Practice

54. **Short Response** Factor the expression $40x + 15$.

55. Which of the following expressions cannot be factored?

A $6 + 3x$
B $7x + 3$
C $15x + 10$
D $30x + 40$

56. The Venn diagram shows the factors of 12 and $18x$.



What is the greatest common factor of the two monomials?

F 2
G 3
H 6
J 36

Common Core Review

Find each product or quotient. 8.EE.1

57. $2^4 \cdot 2^6$

58. $\frac{a^3}{a^{-3}}$

59. $4x^{-2} \cdot 3x^9$

60. $\frac{c^5}{c^9}$

61. $(-4)s^{-8} \cdot (-4)s^7$

62. $\frac{12y^8}{6y^{10}}$

63. Tionne can ride 6 miles on her bike in one hour. If she rode for 1.5 hours on Saturday and 2 hours on Sunday, use mental math to find the total distance she rode that weekend. Justify your answer by using the Distributive Property. 7.NS.2c

64. A commission is a fee paid to a salesperson based on a percent of sales. Suppose a real estate agent earns a 3% commission. What commission would be earned for selling a house for $230,000? 7.RP.3

Add or subtract. 7.EE.1

65. $(-4x + 7) + (5x - 9)$

66. $(4.3x - 2) - (2.2x - 4)$

67. $\left(-\frac{5}{8}x + 3\right) + \left(\frac{3}{4}x - 8\right)$

68. $(6x - 4) - (6x + 1)$