

Jordan and Katelyn collect Matchbox cars. Jordan has 5 more than half as many cars as Katelyn has. Altogether they have 35 cars. How many cars does each person have?

$C = \text{CARS}$



JORDAN 15

KATELYN = C

KATELYN 20

JORDAN = $\frac{1}{2}C + 5$

$$\begin{aligned}
 C + \frac{1}{2}C + 5 &= 35 \\
 \frac{3}{2}C + 5 &= 35 \\
 -5 & \quad -5 \\
 \frac{3}{2}C &= 30 \\
 +5 & \quad +5 \\
 \frac{3}{2}C + 5 &= 35 \\
 \frac{1}{2} & \quad - \frac{1}{2} \\
 \frac{3}{2}C - \frac{1}{2} &= 29\frac{1}{2} \\
 \frac{2}{3} \left(\frac{3}{2}C \right) &= 30 \left(\frac{2}{3} \right) \\
 C &= 20
 \end{aligned}$$



Lesson 8-1

Solving Equations with Rational Coefficients



Interactive Study Guide

See pages 169–170 for:

- Getting Started
- Vocabulary Start-Up
- Notes



Essential Question

How are equations and inequalities used to describe and solve multi-step problems?



Common Core State Standards

Content Standards
7.EE.4, 8.EE.7, 8.EE.7b

Mathematical Practices
1, 3, 4, 5, 7



Vocabulary

solution
inverse operations
equivalent equations

What You'll Learn

- Solve equations by using the Division Property of Equality.
- Solve equations by using the Multiplication Property of Equality.



Real-World Link

Social Networks Do you have a social network profile? While gaming, blogging, and watching videos continue to be popular online activities, more preteens and teens are participating in social networking than ever before. Three-fourths of teens surveyed said they belonged to a social network, compared to 40% of adults surveyed.



Solve Equations by Dividing

An *equation* is a mathematical sentence stating that two expressions are equal. If 72 teens in a survey say they have a social network profile, and these teens are three-fourths of the teens surveyed, then the equation $\frac{3}{4}x = 72$ can be used to find the total number of teens surveyed. A value for the variable that makes an equation true is called a **solution**. For $\frac{3}{4}x = 72$, the solution is 96, since $\frac{3}{4}(96) = 72$ is a true statement.

Key Concept Properties of Equality

Words	Symbols
Multiplication Property of Equality If you multiply each side of an equation by the same nonzero number, the two sides remain equal.	For any numbers a , b , and c , if $a = b$, then $ca = cb$.
Division Property of Equality If you divide each side of an equation by the same nonzero number, the two sides remain equal.	For any numbers a , b , and c , where $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

You can use the above properties and inverse operations to solve an equation.

Inverse operations “undo” each other. To undo the multiplication of $\frac{3}{4}$ in $\frac{3}{4}x = 72$, you can apply the Division Property of Equality to divide each side of the equation by $\frac{3}{4}$. Applying the above properties creates **equivalent equations**, which are equations that have the same solution.



Example 1

Solve $4.2x = -52.5$. Check your solution.

$$4.2x = -52.5 \quad \text{Write the equation.}$$

$$\frac{4.2x}{4.2} = \frac{-52.5}{4.2} \quad \text{Division Property of Equality}$$

$$1x = -12.5 \quad 4.2 \div 4.2 = 1; -52.5 \div 4.2 = -12.5$$

$$x = -12.5 \quad \text{Identity Property; } 1x = x$$

To check your solution, replace x with -12.5 in the original equation.

Check

$$4.2x = -52.5 \quad \text{Write the equation.}$$

$$4.2(-12.5) \stackrel{?}{=} -52.5 \quad \text{Replace } x \text{ with } -12.5.$$

$$-52.5 = -52.5 \checkmark \quad \text{The sentence is true.}$$

The solution is -12.5 .

1b. $-20.9y = -250.8$

$$\frac{-20.9y}{-20.9} = \frac{-250.8}{-20.9}$$

$$y = 12$$

1d. $323.4 = -13.2z$

$$\frac{323.4}{-13.2} = \frac{-13.2z}{-13.2}$$

$$-24.5 = z$$

Vocabulary Link
solution
Everyday Use the answer to a problem, such as the solution to a mystery
Math Use a value of the variable that makes an equation true

1a. $-19.6 = 5.6x$

$$\frac{-19.6}{5.6} = \frac{5.6x}{5.6}$$

$$-3.5 = x$$

Got It? Do these problems to find out.

- 1a. Solve $-19.6 = 5.6x$.
- 1b. Solve $-20.9y = -250.8$.
- 1c. Solve $-13.6a = 217.6$.
- 1d. Solve $323.4 = -13.2z$.



Example 2

A drive-through safari zoo charges \$12.50 per person for admission. In one hour, the park raised \$675 in admission fees. Write and solve an equation to find how many people visited that hour.

Words	Admission fee times the number of visitors equals the total money raised.
Variable	Let v = the number of zoo visitors.
Equation	$12.50 \cdot v = 675$

$$12.5v = 675 \quad \text{Write the equation.}$$

$$\frac{12.5v}{12.5} = \frac{675}{12.5} \quad \text{Division Property of Equality}$$

$$v = 54 \quad \text{Simplify. Check this solution.}$$

The zoo admitted 54 people in one hour.

Got It? Do this problem to find out.

- 2. A one-year, in-state camping permit for New Mexico State Parks costs \$180. If the total income from the camping permits is \$8280 during the first day of sales, write and solve an equation to find how many permits were purchased.

46 PERMITS PURCHASED

$$\frac{(-)}{(-)} = (+)$$

$$\frac{(-)}{(+)} = (-) \quad \frac{(+)}{(-)} = (-)$$

$$\frac{(+)}{(+)} = (+)$$

DIVIDE BY THE COEFFICIENT

PERMITS = P

$$\frac{8280}{180} = \frac{180P}{180}$$

$$46 = P$$



Solve Equations by Multiplying

Equations in which a variable is divided can be solved by multiplying each side by the same number. This method is also useful when the coefficient of the variable is a fraction.

Example 3



Solve each equation. Check your solution.

a. $\frac{1}{4}y = -8$

$$\frac{1}{4}y = -8$$

Write the equation.

$$4 \cdot \frac{1}{4}y = 4 \cdot (-8)$$

Multiplication Property of Equality

$$1y = -32$$

Multiplicative Inverse Property; $4 \cdot \frac{1}{4} = 1$

$$y = -32$$

Identity Property. Check your solution.

b. $-\frac{3}{5}x = -\frac{6}{25}$

$$-\frac{3}{5}x = -\frac{6}{25}$$

Write the equation.

$$-\frac{5}{3} \left(-\frac{3}{5} \right) x = -\frac{5}{3} \left(-\frac{6}{25} \right)$$

Multiply each side by $-\frac{5}{3}$.

$$1x = \frac{2}{5}$$

Multiplicative Inverse Property; $-\frac{5}{3} \left(-\frac{3}{5} \right) = 1$

$$x = \frac{2}{5}$$

Identity Property. Check your solution.

Multiplicative Inverse

Remember that the product of a number and its multiplicative inverse is 1. Use this property when the coefficient of x is a fraction.

Got It? Do these problems to find out.

3a. $7 = -\frac{1}{2}x$

3b. $\frac{6}{7}m = -\frac{3}{14}$

3c. $-9 = \frac{1}{3}n$

3d. $-\frac{1}{4} = \frac{2}{7}y$

Guided Practice



Solve each equation. Check your solution. (Examples 1 and 3)

1. $1.3c = -65$

2. $-4.2 = -7m$

3. $0.8p = 9.6$

4. $\frac{1}{12}n = 12$

5. $18 = \frac{-1}{2}t$

6. $0.6h = 1.8$

7. $-3.4 = 0.4j$

8. $-\frac{3}{4}k = \frac{2}{3}$

9. $\frac{1}{25} = \frac{3}{5}m$



10. A forest preserve rents canoes for \$22.50 per hour. Corey has \$90 to spend. Write and solve an equation to find how many hours he can rent a canoe. (Example 2)



11. **STEM** The weight of an object on the Moon is one-sixth its weight on Earth. If an object weighs 54 pounds on the Moon, write and solve an equation to find how much it weighs on Earth. (Example 2)

Independent Practice

Go online for Step-by-Step Solutions



Solve each equation. Check your solution. (Examples 1 and 3)

12. $9x = 5.4$

13. $0.5s = -60$

14. $6.4 = -0.4r$

15. $-7.2 = 3y$

16. $0.3x = -4.5$

17. $4.95 = 0.3t$

18. $-8.4 = -6g$

19. $-28 = -\frac{1}{14}d$

20. $\frac{1}{9}b = -108$

21. $16 = -\frac{1}{4}b$

22. $-\frac{1}{8}x = -4$

23. $-\frac{4}{9} = -\frac{4}{3}s$

24. $-\frac{25}{36} = -\frac{5}{6}r$

25. $-\frac{9}{10}k = 72$

26. $\frac{2}{3}n = -22$

27. Rashid picked a total of 420 strawberries in $\frac{5}{6}$ hour. Write and solve an equation to find how many strawberries Rashid could pick in 1 hour. (Example 2)

28. **Financial Literacy** Marcus wants to save \$378 in order to buy a new electronic keyboard. He plans to save \$15.75 every week from his paycheck. Write and solve an equation to find how many weeks Marcus will need to save. (Example 2)

Solve each equation. Check your solution.

29. $5p - 2p = -12$

30. $42 = 4x + 3x$

31. $-2(6y) = 144$

32. $72 = -12(-3x)$

33. $\frac{r}{4} = -25 + 9$

34. $\frac{m}{-3} = -5 - 18$

35. $\frac{1}{3}n = \frac{2}{9}$

36. $\frac{5}{8} = -\frac{1}{2}x$

37. $-0.7 = -\frac{7}{9}z$

38. $1\frac{7}{8}y = 4\frac{1}{2}$

39. $2\frac{1}{3} = -9m$

40. $-\frac{7}{9}t = -\frac{28}{36}$

41. The sleeping heart rate of a black bear during hibernation is about $\frac{2}{5}$ of its summer rate. If the sleeping heart rate of a bear is 28 beats per minute during hibernation, find the summer sleeping heart rate.

42. Use the table to help you write and solve an equation.

- Jenna hikes at a steady rate of 2.5 miles per hour. How long will it take her to hike the Lone Star Trail?
- It takes Ming 4 hours to hike the Mallard Lake Trail. What is his average hiking rate?
- The round-trip distance of the Observation Point Trail is 56% of the round-trip distance of the Mystic Falls Trail. What is the round-trip distance of the Mystic Falls Trail?

Yellowstone National Park, Wyoming	
Name of Trail	Round-Trip Distance (mi)
Mallard Lake	6.8
Howard Eaton	5.8
Lone Star	4.8
Observation Point	1.4

43. **CCSS Use Math Tools** A student solved the equation $\frac{9}{10}m = 5\frac{1}{10}$ and found the solution $m = 459$. Use estimation to explain why the student's solution must be incorrect.

44. **CCSS Identify Structure** Explain how to use the Multiplication Property of Equality to solve $\frac{2}{5}y = 6$. Then explain how to solve the equation using the Division Property of Equality.

$$29. \quad 5p - 2p = -12$$

$$\frac{3p}{3} = \frac{-12}{3}$$

$$p = -4$$

$$5(-4) - (2)(-4) = -12$$

$$-20 - (-8)$$

$$-12 = -12$$

$$39. 2\frac{1}{3} = -9m$$

$$\frac{7}{3} = \frac{-9m}{\frac{7}{3}}$$

$$\frac{7}{3} = \frac{-9m}{-9m}$$

$$\frac{7}{3} = 1$$

$$40. -\frac{7}{9}t = -\frac{28}{36}$$

$$\frac{7}{3} = \frac{-9m}{-9}$$

$$\frac{7}{3} = m$$

$$\frac{7}{3} \left(-\frac{1}{9}\right) = m$$

$$-\frac{7}{27} = m$$

$$\frac{7}{3} = \frac{1}{1} \left(-\frac{7}{27}\right)$$

$$\frac{7}{3} = \frac{7}{3}$$

$$-\frac{7}{9}t = -\frac{28}{36}$$

$$-\frac{9}{7} \left(-\frac{7}{9}t\right) = \left(-\frac{7}{9}\right) \left(-\frac{28}{36}\right)$$

$$t = 1$$

Solve each equation. Check your solution.

29. $5p - 2p = -12$

30. $42 = 4x + 3x$

31. $-2(6y) = 144$

32. $72 = -12(-3x)$

33. $\frac{r}{4} = -25 + 9$

34. $\frac{m}{-3} = -5 - 18$ $\frac{1}{4}r = -16$

31. $-2(6y) = 144$

$$\frac{-12y}{-12} = \frac{144}{-12}$$

$$y = -12$$

CHECK $-2(6 \cdot -12) = 144$

$$-2(-72)$$

$$144 = 144$$

✓

34. $\frac{m}{-3} = -5 - 18$

$$\frac{m}{-3} = -5 + (-18)$$

$$-3\left(\frac{m}{-3}\right) = (-23)(-3)$$

$$m = 69$$

$$\frac{69}{-3} = -23$$

$$-23 = -23$$

✓

33. $\frac{r}{4} = -25 + 9$

$$4\left(\frac{r}{4}\right) = (-16)4$$

$$r = -64$$

$$\frac{-64}{4} = -16$$

$$-16 = -16$$

✓

Solve each equation. Check your solution.

29. $5p - 2p = -12$

30. $42 = 4x + 3x$

31. $-2(6y) = 144$

32. $72 = -12(-3x)$

33. $\frac{r}{4} = -25 + 9$

34. $\frac{m}{-3} = -5 - 18$

$\frac{20}{5} = 4$

SIMPLIFY BEFORE SOLVING

33. $\frac{r}{4} = -25 + 9$

Times 4
to cancel
divide by 4
 $4 \left(\frac{r}{4} \right) = (-16) 4$
 $r = -64$

$\frac{-64}{4} = -16$

34. $\frac{m}{-3} = -5 - 18$

$-3 \left(\frac{m}{-3} \right) = -23(-3)$

$m = 69$

$\frac{69}{-3} = -23$

$-23 = -23$
✓

27. Rashid picked a total of 420 strawberries in $\frac{5}{6}$ hour. Write and solve an equation to find how many strawberries Rashid could pick in 1 hour. (Example 2)

$$400 \text{ STRAWBERRIES IN } \frac{1}{2} \text{ HOUR} = 800 \text{ STRAWBERRIES / HOUR}$$

$$400 \div \frac{1}{2} = 400 \cdot 2 = 800$$

$$\frac{420}{\frac{5}{6}} = 420 \cdot \frac{6}{5} = \frac{2520}{5} = 504$$

$$\frac{420}{\frac{1}{6}} = 420 \cdot \frac{6}{1} = 2520$$

45 **CCSS** **Multiple Representations** Every autumn, the North American Monarch butterfly migrates up to 3000 miles to California and Mexico, where it hibernates until early spring. Suppose a particular butterfly travels on average 52.5 miles per day.

a. Symbols Write an equation that represents the distance d the butterfly will travel in t days.

b. Table Use the equation to complete the table.

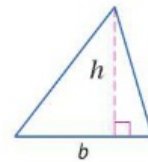
Time (days)	1	2	3	4	5	6
Distance (miles)	■	■	■	■	■	■

c. Graph Graph the points from the table on the coordinate plane. Graph time on the x -axis and distance on the y -axis.

d. Graph Using the graph, estimate the number of days it will take the butterfly to travel 475 miles.

e. Words How many days will it take the butterfly to travel 2100 miles? Which method did you use to solve the problem?

46. The formula for finding the area of a triangle is $A = \frac{1}{2}bh$, where A represents the area, b represents the length of the base of the triangle, and h represents the height of the triangle. Write and solve equations to complete the table of values.



Area (A)	15	15	15	15	15
Base (b)	1	2	3	4	5
Height (h)	■	■	■	■	■



H.O.T. Problems Higher Order Thinking

47. CCSS Model with Mathematics Write a real-world example that uses an equation containing a decimal and a fraction. Then find the solution.

48. CCSS Find the Error Sam is solving $\frac{1}{4}x = -20$. Find his mistake and correct it.

$$\frac{1}{4}x \div 4 = -20 \div 4$$

$$x = -5$$

49. CCSS Justify Conclusions When you solve an equation of the form $px = q$, where p and q are rational numbers and $p \neq 0$, is it ever possible for the solution to be 0? If so, when? If not, why not? Justify your conclusion.

$$px = 0$$


$$px = q$$

$$p(0) = 0$$

50. CCSS Persevere with Problems If $\frac{3}{10}x = 3$, what is the value of $7x + 13$?

51. e Building on the Essential Question Suppose your friend says he can solve $3x = 15$ by using the Multiplication Property of Equality. Is he correct? Justify your response.


Standardized Test Practice

52. During a vacation, the Morales family drove 63.2 miles in 1 hour. If they averaged the same speed during their trip, which equation can be used to find how far the Morales family drove in 6 hours?
- A $\frac{63.2}{x} = 6$ C $6x = 63.2$
 B $\frac{1}{6}x = 63.2$ D $63.2x = 6$
53. The solution of which equation is *not* graphed on the number line below?
- 
- F $12.8 = -6.4x$
 G $8.1x = -16.2$
 H $-15 = 7.5x$
 J $-18.3x = -36.6$
54. Ella paid \$11.85 for 3 magazines. If each magazine was the same price, how much did each magazine cost?
- A \$4.59
 B \$4.00
 C \$3.95
 D \$3.59
55. **Short Response** Stanley paid half of what Royce paid for his baseball glove. Royce paid \$64.98 for his glove. Write and solve an equation to find how much Stanley paid for his glove.


Common Core Review

Solve each equation. Check your solution. 8.EE.7

56. $x - 5 = -22$ 57. $4 = 7 + p$ 58. $-40 = y - 9$
 59. $2.3 + r = 1.6$ 60. $d - 2.7 = -1.4$ 61. $t + (-16) = -24$
 62. $p + \frac{1}{10} = -\frac{3}{4}$ 63. $\frac{2}{3} + k = \frac{1}{6}$ 64. $d - \frac{4}{9} = -\frac{1}{12}$

Simplify each expression. 6.EE.3

65. $5(t + 3)$ 66. $7x - 12x$ 67. $9p + 4 + 3p$
 68. $3w + 4s - w + 5s$ 69. $7 - 4(x + 3)$ 70. $3(2 + 3x) + 21x$
 71. $12x + 6 + x$ 72. $4n - 9 + 5n$ 73. $10(y + 8)$

74. Find the values that complete the table at the right for $y = -4x$. 7.NS.2

75. Gabriel is 12 years old, and his younger brother Elias is 2 years old. How old will each of them be when Gabriel is twice as old as Elias? 7.EE.3

x	-2	-1	0	1
y	■	■	■	■

Find the value of each expression. 6.EE.1

76. $3^3 + 7(4)$ 77. $\frac{2^2 - 9}{10 + 15}$ 78. $5 - 3(6 + 2^3)$
 79. $5^2 - 6 + 6 \cdot 8$ 80. $20 \cdot 7 - 2^4 \cdot 5$ 81. $3^2[15 - (-9)]$
 82. $6^2 - 4^2 \cdot 3$ 83. $5(9^2 - 10^2)$ 84. $-4^3 + 2 \cdot 9$
 85. $10 - 5(10 - 8)^2$ 86. $(4 + 3^3)(-2)$ 87. $\frac{(1 - 3)^2}{(4 - 2)^2}$

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