Lesson 1-4
Properties of Numbers

<dInteractive Study Guide

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- Getting Started
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Essential Question

How can you use numbers and symbols to represent mathematical ideas?

Common Core State Standards

Content Standards
7.EE.1, 7.EE.2

Mathematical Practices
1, 3, 4, 7

Vocabulary

properties
Commutative Property
Associative Property
counterexample
simplify
deductive reasoning

$10 \cdot (42 \cdot 7) = (10 \cdot 42) \cdot 7$

$5 \cdot (2 \cdot 19) = (5 \cdot 2) \cdot 19$

Jeff Greenberg/Alamy

What You'll Learn

- Identify and use properties of addition and multiplication.
- Use properties to simplify algebraic expressions.

Real-World Link

Crafts Duct tape has been used to create everything from flip-flops and prom dresses to wallets and homemade flotation devices. Algebraic properties can be used to find the amount of duct tape needed to make an item.

Properties of Addition and Multiplication

In algebra, **properties** are statements that are true for any numbers. For example, the expressions 30 + 10 and 10 + 30 have the same value, 40. This illustrates the **Commutative Property of Addition**. Likewise, 30 · 10 and 10 · 30 have the same value, 300. This illustrates the **Commutative Property of Multiplication.**

Key Concept: Commutative Properties

Words The order in which numbers are added or multiplied does not change the sum or product.

Symbols For any numbers a and b, $a + b = b + a$.
For any numbers a and b, $a \cdot b = b \cdot a$.

Examples

$6 + 9 = 9 + 6$ $\qquad$ $4 \cdot 7 = 7 \cdot 4$
$15 = 15$ $\qquad$ $28 = 28$

$2 \cdot 3 = 3 \cdot 2$
$6 \quad 6$

$5+2 = 2+5$
$7 \quad 7$

ORDER PROPERTY

To evaluate the expression 16 + (14 + 58), use mental math by grouping the numbers as (16 + 14) + 58 since 4 + 6 = 10. This illustrates the **Associative Property of Addition**. There is also an **Associative Property of Multiplication**.

Key Concept: Associative Properties

Words The way in which numbers are grouped when added or multiplied does not change the sum or product.

Symbols For any numbers a, b, and c, $(a + b) + c = a + (b + c)$.
For any numbers a, b, and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Examples

$(3 + 6) + 1 = 3 + (6 + 1)$ $\qquad$ $(5 \cdot 9) \cdot 2 = 5 \cdot (9 \cdot 2)$
$9 + 1 = 3 + 7$ $\qquad$ $45 \cdot 2 = 5 \cdot 18$
$10 = 10$ $\qquad$ $90 = 90$

GROUPING PROPERTY

In addition to the Commutative and Associative Properties, the Identity and Zero Properties are also true for any numbers.

Identity
The word identity means *sameness of essential character*. The additive identity, 0, and multiplicative identity, 1, allow the original number to remain the same.

Key Concept Number Properties

Property	Words	Symbols	Examples
Additive Identity	When 0 is added to any number, the sum is the number.	For any number a, $a + 0 = 0 + a = a$	$5 + 0 = 5$ $0 + 5 = 5$
Multiplicative Identity	When any number is multiplied by 1, the product is the number.	For any number a, $a \cdot 1 = 1 \cdot a = a$.	$8 \cdot 1 = 8$ $1 \cdot 8 = 8$
Multiplicative Property of Zero	When any number is multiplied by 0, the product is 0.	For any number a, $a \cdot 0 = 0 \cdot a = 0$	$3 \cdot 0 = 0$ $0 \cdot 3 = 0$

$-5 + 1.25 + 5$

$\frac{4}{6} \cdot \frac{\boxed{5}}{\boxed{5}} = \frac{20}{30}$

$\frac{9}{12} \div \frac{\boxed{3}}{\boxed{3}} = \frac{3}{4}$

Do these properties apply to subtraction or division? One way to find out is to look for a counterexample. A **counterexample** is an example that shows a statement is not true.

Example 1

Is division of whole numbers associative? If not, give a counterexample.

The Associative Property of Multiplication states $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. To determine whether the Associative Property applies to division, check $(a \div b) \div c \stackrel{?}{=} a \div (b \div c)$.

$(27 \div 9) \div 3 \stackrel{?}{=} 27 \div (9 \div 3)$ Pick values for a, b, and c.

$(3) \div 3 \stackrel{?}{=} 27 \div (3)$ Simplify.

$1 \neq 9$ Simplify.

We found a counterexample. So, division of whole numbers is not associative.

Counterexample
You can disprove a statement by finding only one counterexample.

Got It? Do this problem to find out.

1. Is subtraction of decimals associative? If not, give a counterexample.

$(10 \div 2) \div 2 = 2.5$

$10 \div (2 \div 2) = 10$

$10 \div 5 = 2$

$5 \div 10 = \frac{1}{2}$

Example 2

Name the property shown by each statement.

a. $4 + (a + 3) = (a + 3) + 4$

The order of the numbers and variables changed. This is the Commutative Property of Addition.

b. $1 \cdot (3c) = 3c$

The expression was multiplied by 1 and remained the same. This is the Multiplicative Identity Property.

Got It? Do these problems to find out.

2a. $d + 0 = d$

2b. $8 \cdot 1 = 8$

2c. $14 + (9 + 10) = (14 + 9) + 10$

2d. $5 \times 7 \times 2 = 7 \times 2 \times 5$