Lesson 1-3

Variables and Expressions

ISG Interactive Study Guide

See pages 9–10 for:
• Getting Started
• Real-World Link
• Notes

Essential Question

How can you use numbers and symbols to represent mathematical ideas?

CCSS Common Core State Standards

Content Standards
7.NS.3, 7.EE.4

Mathematical Practices
1, 3, 4

Vocabulary

algebra
variable
algebraic expression
defining a variable
Substitution Property of Equality

Handwritten notes (left margin):
$5(15)$
$16(15)$
$8(15)$
$h = \text{NUMBER OF HOURS}$
$15h$

What You'll Learn

• Translate verbal phrases into algebraic expressions.
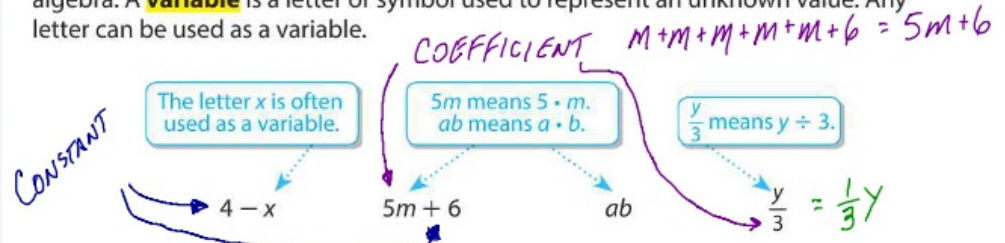• Evaluate expressions containing variables.

Real-World Link

Online Games Did you know that nearly 250 million people worldwide participate in online gaming? Mathematical expressions can be used to represent the total points earned by a player in an online game.

Algebraic Expressions and Verbal Phrases

Algebra is a branch of mathematics that uses symbols. A variable is often used in algebra. A **variable** is a letter or symbol used to represent an unknown value. Any letter can be used as a variable.

Handwritten note:
COEFFICIENT $\quad m + m + m + m + m + 6 = 5m + 6$

CONSTANT

| The letter x is often used as a variable. | $5m$ means $5 \cdot m$. ab means $a \cdot b$. | $\frac{y}{3}$ means $y \div 3$. |

$$4 - x \qquad 5m + 6 \qquad ab \qquad \frac{y}{3} = \frac{1}{3}y$$

An expression like $5m + 6$ is an **algebraic expression** because it contains at least one variable and at least one mathematical operation.

The first step in translating verbal phrases into algebraic expressions is to choose a variable and a quantity for the variable to represent. This is called **defining a variable**. All of the steps involved in writing algebraic expressions are shown below.

Words	Describe the situation. Use only the most important words.
Variable	Define a variable by choosing a variable to represent the unknown quantity.
Expression	Translate your verbal model into an algebraic expression.

Example 1

Translate each phrase into an algebraic expression.

a. three dollars more than the cost of a sandwich

Words	three dollars more than the cost of a sandwich
Variable	Let c represent the cost of a sandwich.
Expression	$3 + c$

Handwritten notes:
$3(2) = 6$
$2(3) = 6$
$3 + 2 = 2 + 3$ COMMUTATIVE
$10 - 6 = 4$ OPPOSITES
$6 - 10 = -4$

b. Mari had $2 and made $6 an hour babysitting.

Words	two more than six dollars per hour
Variable	Let n represent the number of hours.
Expression	$2 + 6n$

Handwritten notes:
$\frac{10}{5} = \frac{2}{1}$ RECIPROCALS
$\frac{5}{10} = \frac{1}{2}$
$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$

Products with Variables

When you write the product of a number and a variable, you should always write the number first. Write $2 + 6n$ rather than $2 + n6$.

Got It? Do these problems to find out.

1a. two miles less than the athlete ran *a = MILES THE ATHLETE*   *a - 2*

1b. five points more than the points scored by field goals if each field goal is worth 3 points   *f = number of field*   *3f + 5*

Key Concept Substitution Property of Equality

Words	If two quantities are equal, then one quantity can be replaced by the other.
Symbols	For all numbers a and b, if $a = b$, then a may be replaced by b.

To evaluate an algebraic expression, replace the variable(s) with known values and then use the order of operations. When you replace a variable with a number, you are using the **Substitution Property of Equality**.

Example 2

Evaluate $d + 5 - f$ if $d = 16$ and $f = 18$.

$d + 5 - f = 16 + 5 - 18$ Replace d with 16 and f with 18.

$\qquad\quad = 21 - 18$ Add 16 and 5.

$\qquad\quad = 3$ Subtract 18 from 21.

Got It? Do these problems to find out.

2a. Evaluate $6 - e + f$ if $e = 3$ and $f = 9$.   *6 - 3 + 9 = 3 + 9 = 12*

2b. Evaluate $7k + h$ if $k = 4$ and $h = 10$.   *7(4) + 10 = 28 + 10 = 38*

Example 3

Tutor

Evaluate each expression if $r = 1$, $s = 5$, and $t = 8$.

a. $6s + 2t$

$$6s + 2t = 6(5) + 2(8) \qquad \text{Replace } s \text{ with 5 and } t \text{ with 8.}$$
$$= 30 + 16 \text{ or } 46 \qquad \text{Multiply. Then add.}$$

b. $\dfrac{st}{20}$

$$\frac{st}{20} = st \div 20 \qquad \text{Rewrite as a division expression.}$$
$$= (5 \cdot 8) \div 20 \qquad \text{Replace } s \text{ with 5 and } t \text{ with 8.}$$
$$= 40 \div 20 \text{ or } 2 \qquad \text{Multiply. Then divide.}$$

c. $r + (40 - 3t)$

$$r + (40 - 3t) = 1 + (40 - 3 \cdot 8) \qquad \text{Replace } r \text{ with 1 and } t \text{ with 8.}$$
$$= 1 + (40 - 24) \qquad \text{Multiply 3 and 8.}$$
$$= 1 + 16 \text{ or } 17 \qquad \text{Subtract 24 from 40. Then add 1 and 16.}$$

> ⚠️ **Watch Out!**
>
> In Example 3a, use parentheses when you replace the variables with numbers. This will help you remember to multiply.

Got It? Do these problems to find out.

Evaluate each expression if $a = 4$, $b = 8$, and $c = 12$.

3a. $3a + 2c$ **3b.** $\dfrac{ab}{16}$ **3c.** $c + (5b - 2a)$

Example 4

Tutor

A company rents a houseboat for $200 plus an extra $30 per day.

a. Write an expression that can be used to find the total cost to rent a houseboat.

Words	two-hundred-dollar rental fee plus thirty dollars per day
Variable	Let d represent the number of days.
Expression	$200 + 30d$

The expression is $200 + 30d$.

b. Suppose the Gregoran family wants to rent a houseboat for six days. What will be the total cost?

$$200 + 30d = 200 + 30(6) \qquad \text{Replace } d \text{ with 6.}$$
$$= 200 + 180 \text{ or } 380 \qquad \text{Multiply. Then add.}$$

The total cost will be $380.

Got It? Do this problem to find out.

4. At a garage sale, each DVD was marked at $5, and each CD was marked at $3. Write an expression to find the total cost to buy some DVDs and CDs. Then find the cost of buying 4 DVDs and 7 CDs.

Guided Practice

Translate each phrase into an algebraic expression. (Example 1)

1. four dollars less than the cost of the sweater

2. thirteen more students than teachers

3. money earned babysitting at $10 per hour

4. thirty pencils divided among some students

Evaluate each expression if $g = 6$, $h = 10$, and $j = 5$. (Examples 2 and 3)

5. $h + 15$

6. $g - 3$

7. $20 - h + g$

8. $22 - 3j$

9. $\dfrac{gh}{j}$

10. $4g + (3h - 4j)$

11. One pint of liquid is the same as 16 fluid ounces. (Example 4)

    a. Suppose the number of pints of liquid is represented by p. Write an expression to find the number of fluid ounces.

    b. How many fluid ounces are in 5 pints?

Independent Practice

eHelp

Translate each phrase into an algebraic expression. (Example 1)

12. three times as many balloons

13 twenty-four pieces of candy divided among some students

14. the number of people increased by thirteen

15. the number of inches in any number of feet

16. four more than the number of weeks in a group of days

17. four less than the amount of cents in a number of dimes

Evaluate each expression if $a = 9$, $b = 4$, and $c = 11$. (Examples 2 and 3)

18. $b + 9$

19. $13 - a$

20. $2c - 5$

21. $18 + 4b$

22. $\dfrac{ab}{6}$

23. $\dfrac{8a}{b}$

24. $5c - 4a$

25. $7b - 2c$

26. $45 - \dfrac{bc}{2}$

27. $\dfrac{ac}{3} - 15$

28. $4b + 3c - 5a$

29. $6c - 2a + 6b$

30. A studio charges a sitting fee of $25 plus $7 for each portrait sheet ordered. Write an expression that can be used to find the total cost to have photographs taken. Then find the cost of purchasing twelve portrait sheets. (Example 4)

31. One gallon of water is equal to 231 cubic inches. Write an expression for the number of gallons of water in any number of cubic inches of water. (Example 4)

Evaluate each expression if $x = 9$, $y = 4$, and $z = 12$.

32. $7z - (y + x)$

33. $(8y + 5) - 2z$

34. $(5z - 4x) + 3y$

35. $6x - (z - 2y)$

36. $2x + (4z - 13) - 5$

37. $(29 - 3y) + 4z - 7$

38. Financial Literacy A cell phone company offers two different monthly plans. Plan A costs a flat rate of $0.10 per minute for all calls. Plan B costs $29.99 for the first 500 minutes and $0.08 for each additional minute. Which plan is less costly if a person uses 750 minutes per month? Explain.

39 One bushel of apples from a dwarf apple tree is equal to 42 pounds. Write an expression to find the number of pounds of apples in any number of bushels. If one tree can produce 6 bushels, how many pounds of apples will an orchard of 100 trees produce?

40. A car rental company charges a one-time fee of $95, plus $65 per day. Write an expression that can be used to find the cost of renting a car for any number of days. If George rents a car for 4 days, what is the total cost?

41. CCSS Multiple Representations In this problem, you will use algebra to describe a relationship. Jacinda used the table below to help convert measurements while she was cooking.

Number of Cups (c)	4	8	12	16
Number of Quarts (q)	1	2	3	4

a. Words Write an expression in words that describes the relationship between the number of quarts and the number of cups.

b. Symbols Write an algebraic expression that represents the number of quarts in c cups.

c. Numbers Use the expression in part **b** to find the number of quarts in 100 cups.

🔥 H.O.T. Problems Higher Order Thinking

42. CCSS Model with Mathematics Write an algebraic expression that has two different variables and two different operations: addition, subtraction, multiplication, or division. Then write a real-world problem that uses the expression.

43. CCSS Find the Error John is writing an algebraic expression for the phrase *five less than a number*. Find his mistake and correct it.

> Let n represent the number.
> $5 - n$

44. CCSS Persevere with Problems Franco constructed the objects below using toothpicks.

Figure 1   Figure 2   Figure 3

Write two different rules that relate the figure number to the number of toothpicks in each figure. Explain how you arrived at your answers.

45. ℮ Building on the Essential Question Cassandra needs to evaluate the expression $a(x + y)$. After she replaces the variables with numerical values, in which order should she perform the operations of addition and multiplication? Explain.

46. What word phrase is equivalent to the expression $5x + 9$?

 A five cents more than nine nickels

 B nine cents plus five cents

 C nine cents more than five nickels

 D nine cents less than five nickels

47. Which rule describes the ordered pairs in this table?

x	y
1	1
2	4
3	7
4	10

 F $y = x$

 G $y = 2x$

 H $y = 3x - 2$

 J $y = 2x + 2$

48. What is "8 more than the quotient of five and a number n" written as an algebraic expression?

 A $5 + 8n$

 B $8 + 5n$

 C $8 + 5 \div n$

 D $5 + n \div 8$

49. **Short Response** The table below shows how much Ava will pay to rent one DVD and one game for the number of days given.

Number of Days	Total Cost ($)
2	10
4	20
6	30

How much will Ava pay to rent one DVD and one game for 7 days?

Common Core Review

Find the value of each expression. 7.NS.3

50. $3 \cdot 6 - 4$

51. $12 - 3 \times 3$

52. $9 + 18 \div 3$

53. $56 \div (7 \cdot 2) \times 6$

54. $75 \div (7 + 8) - 3$

55. $70 - (16 \div 2 + 21)$

56. $\dfrac{45 - 18}{9 \div 3}$

57. $\dfrac{8 \div 8 + 11}{15 - 4(3)}$

58. $4(20 - 13) + 4 \times 5$

The final standings of a hockey league are shown. A win is worth 3 points, and a tie is worth 1 point. Zero points are given for a loss. 7.NS.3

Team	Wins	Losses	Ties
Knights	14	9	7
Huskies	11	9	10
Wildcats	10	9	11
Mustangs	9	10	11
Panthers	10	14	6

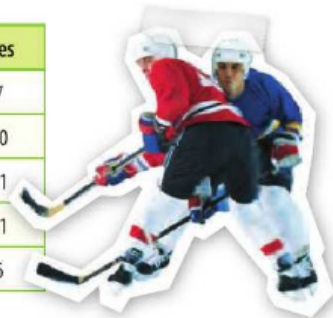59. How many points do the Wildcats have?

60. How many points do the Huskies have?

61. How many more points do the Knights have than the Panthers?

62. How many fewer points do the Mustangs have than the Huskies?

63. At the end of the season, all teams with more than 40 points go to the playoffs. Which teams from the league will go to the playoffs?

Find the least common multiple (LCM) of each pair of numbers. 6.NS.4

64. 6 and 8

65. 5 and 7

66. 6 and 10

67. 4 and 8

68. 1 and 9

69. 3 and 12

70. 10 and 12

71. 9 and 12

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Guided Practice

Translate each phrase into an algebraic expression. (Example 1)

1. four dollars less than the cost of the sweater $c - 4$

2. thirteen more students than teachers $13 + t$

3. money earned babysitting at $10 per hour $10h$

4. thirty pencils divided among some students $30 \div s$

Evaluate each expression if $g = 6$, $h = 10$, and $j = 5$. (Examples 2 and 3)

5. $h + 15$ **25**

6. $g - 3$ **3**

7. $20 - h + g$ **16**

8. $22 - 3j$ **7**

9. $\dfrac{gh}{j}$ **12**

10. $4g + (3h - 4j)$ **34**

11. One pint of liquid is the same as 16 fluid ounces. (Example 4)

   a. Suppose the number of pints of liquid is represented by p. Write an expression to find the number of fluid ounces. **16p**

   b. How many fluid ounces are in 5 pints? **80 fl oz**

Independent Practice

Translate each phrase into an algebraic expression. (Example 1)

12. three times as many balloons $3b$

13. twenty-four pieces of candy divided among some students $24 \div s$

14. the number of people increased by thirteen $p + 13$

15. the number of inches in any number of feet $12n$

16. four more than the number of weeks in a group of days $\dfrac{d}{7} + 4$

17. four less than the amount of cents in a number of dimes $10n - 4$

Evaluate each expression if $a = 9$, $b = 4$, and $c = 11$. (Examples 2 and 3)

18. $b + 9$ **13**

19. $13 - a$ **4**

20. $2c - 5$ **17**

21. $18 + 4b$ **34**

22. $\dfrac{ab}{6}$ **6**

23. $\dfrac{8a}{b}$ **18**

24. $5c - 4a$ **19**

25. $7b - 2c$ **6**

26. $45 - \dfrac{bc}{2}$ **23**

27. $\dfrac{ac}{3} - 15$ **18**

28. $4b + 3c - 5a$ **4**

29. $6c - 2a + 6b$ **72**

30. A studio charges a sitting fee of $25 plus $7 for each portrait sheet ordered. Write an expression that can be used to find the total cost to have photographs taken. Then find the cost of purchasing twelve portrait sheets. (Example 4) $25 + 7s$; **$109**

31. One gallon of water is equal to 231 cubic inches. Write an expression for the number

PG. 17

38. Financial Literacy A cell phone company offers two different monthly plans. Plan A costs a flat rate of $0.10 per minute for all calls. Plan B costs $29.99 for the first 500 minutes and $0.08 for each additional minute. Which plan is less costly if a person uses 750 minutes per month? Explain. **See Answer Appendix.**

39 One bushel of apples from a dwarf apple tree is equal to 42 pounds. Write an expression to find the number of pounds of apples in any number of bushels. If one tree can produce 6 bushels, how many pounds of apples will an orchard of 100 trees produce? **42b; 100 • 42 • 6 or 25,200 lb**

40. A car rental company charges a one-time fee of $95, plus $65 per day. Write an expression that can be used to find the cost of renting a car for any number of days. If George rents a car for 4 days, what is the total cost? **95 + 65d; $355**

41. **Multiple Representations** In this problem, you will use algebra to describe a relationship. Jacinda used the table below to help convert measurements while she was cooking.

Number of Cups (c)	4	8	12	16
Number of Quarts (q)	1	2	3	4

a. **Words** Write an expression in words that describes the relationship between the number of quarts and the number of cups. **There are four cups in one quart.**

b. **Symbols** Write an algebraic expression that represents the number of quarts in c cups. **$c \div 4$**

c. **Numbers** Use the expression in part **b** to find the number of quarts in 100 cups. **25 qt**

H.O.T. Problems Higher Order Thinking

42. **Model with Mathematics** Write an algebraic expression that has two different variables and two different operations: addition, subtraction, multiplication, or division. Then write a real-world problem that uses the expression. **42–45. See Answer Appendix.**

43. **Find the Error** John is writing an algebraic expression for the phrase *five less than a number*. Find his mistake and correct it.

> Let n represent the number.
> $5 - n$

44. **Persevere with Problems** Franco constructed the objects below using toothpicks.

Figure 1   Figure 2   Figure 3

PG. 18

46. What word phrase is equivalent to the expression $5x + 9$? **C**

  A five cents more than nine nickels

  B nine cents plus five cents

  C nine cents more than five nickels

  D nine cents less than five nickels

47. Which rule describes the ordered pairs in this table? **H**

x	y
1	1
2	4
3	7
4	10

  F $y = x$ **H** $y = 3x - 2$

  G $y = 2x$ **J** $y = 2x + 2$

48. What is "8 more than the quotient of five and a number *n*" written as an algebraic expression? **C**

  A $5 + 8n$

  B $8 + 5n$

  C $8 + 5 \div n$

  D $5 + n \div 8$

49. **Short Response** The table below shows how much Ava will pay to rent one DVD and one game for the number of days given.

Number of Days	Total Cost ($)
2	10
4	20
6	30

How much will Ava pay to rent one DVD and one game for 7 days? **$35**

CCSS **Common Core Review**

Find the value of each expression. 7.NS.3

50. $3 \cdot 6 - 4$ **14**

51. $12 - 3 \times 3$ **3**

52. $9 + 18 \div 3$ **15**

53. $56 \div (7 \cdot 2) \times 6$ **24**

54. $75 \div (7 + 8) - 3$ **2**

55. $70 - (16 \div 2 + 21)$ **41**

56. $\dfrac{45 - 18}{9 \div 3}$ **9**

57. $\dfrac{8 \div 8 + 11}{15 - 4(3)}$ **4**

58. $4(20 - 13) + 4 \times 5$ **48**

The final standings of a hockey league are shown. A win is worth 3 points, and a tie is worth 1 point. Zero points are given for a loss. 7.NS.3

Team	Wins	Losses	Ties
Knights	14	9	1
Huskies	11	9	10
Wildcats	10	9	11
Mustangs	9	10	11
Panthers	10	14	6

59. How many points do the Wildcats have? **41 points**

60. How many points do the Huskies have? **43 points**

61. How many more points do the Knights have than the Panthers? **13 points**

62. How many fewer points do the Mustangs have than the Huskies? **5**

63. At the end of the season, all teams with more than 40 points go to the playoffs. Which teams from the league will go to the playoffs? **Knights, Huskies, Wildcats**