Lesson 1-4 Properties of Numbers

Interactive Study Guide

See pages 11–12 for:
- Getting Started
- Real-World Link
- Notes

What You'll Learn
- Identify and use properties of addition and multiplication.
- Use properties to simplify algebraic expressions.

Real-World Link

Crafts Duct tape has been used to create everything from flip-flops and prom dresses to wallets and homemade flotation devices. Algebraic properties can be used to find the amount of duct tape needed to make an item.

Essential Question

How can you use numbers and symbols to represent mathematical ideas?

Common Core State Standards

Content Standards
7.EE.1, 7.EE.2

Mathematical Practices
1, 3, 4, 7

Vocabulary

properties
Commutative Property
Associative Property
counterexample
simplify
deductive reasoning

Properties of Addition and Multiplication

In algebra, **properties** are statements that are true for any numbers. For example, the expressions 30 + 10 and 10 + 30 have the same value, 40. This illustrates the **Commutative Property of Addition**. Likewise, 30 • 10 and 10 • 30 have the same value, 300. This illustrates the **Commutative Property of Multiplication.**

$5 \cdot 13 \cdot 2 = 5 \cdot 2 \cdot 13 = 10 \cdot 13 = 130$

Key Concept Commutative Properties

Words	The order in which numbers are added or multiplied does not change the sum or product.	
Symbols	For any numbers a and b, $a + b = b + a$. For any numbers a and b, $a \cdot b = b \cdot a$.	
Examples	$6 + 9 = 9 + 6$ $15 = 15$	$4 \cdot 7 = 7 \cdot 4$ $28 = 28$

COMMUTATIVE PROPERTY SAYS THAT ORDER DOES NOT MATTER.

To evaluate the expression 16 + (14 + 58), use mental math by grouping the numbers as (16 + 14) + 58 since 4 + 6 = 10. This illustrates the **Associative Property of Addition**. There is also an **Associative Property of Multiplication**.

$(125 + 49) + 51 = 125 + (49 + 51) = 125 + 100 = 225$

Key Concept Associative Properties

Words	The way in which numbers are grouped when added or multiplied does not change the sum or product.	
Symbols	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$. For any numbers a, b, and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.	
Examples	$(3 + 6) + 1 = 3 + (6 + 1)$ $9 + 1 = 3 + 7$ $10 = 10$	$(5 \cdot 9) \cdot 2 = 5 \cdot (9 \cdot 2)$ $45 \cdot 2 = 5 \cdot 18$ $90 = 90$

THE ASSOCIATIVE PROPERTY SAYS GROUPING DOES NOT MATTER

Jeff Greenberg/Alamy

In addition to the Commutative and Associative Properties, the Identity and Zero Properties are also true for any numbers.

Identity
The word identity means *sameness of essential character*. The additive identity, 0, and multiplicative identity, 1, allow the original number to remain the same.

Key Concept Number Properties

Property	Words	Symbols	Examples
Additive Identity	When 0 is added to any number, the sum is the number.	For any number a, $a + 0 = 0 + a = a$	$5 + 0 = 5$ $0 + 5 = 5$
Multiplicative Identity	When any number is multiplied by 1, the product is the number.	For any number a, $a \cdot 1 = 1 \cdot a = a$.	$8 \cdot 1 = 8$ $1 \cdot 8 = 8$
Multiplicative Property of Zero	When any number is multiplied by 0, the product is 0.	For any number a, $a \cdot 0 = 0 \cdot a = 0$	$3 \cdot 0 = 0$ $0 \cdot 3 = 0$

Do these properties apply to subtraction or division? One way to find out is to look for a counterexample. A **counterexample** is an example that shows a statement is not true.

$\frac{5}{7} \cdot \frac{2}{2}, \frac{10}{14}$

Counterexample
You can disprove a statement by finding only one counterexample.

Example 1

Tutor

Is division of whole numbers associative? If not, give a counterexample.

The Associative Property of Multiplication states $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. To determine whether the Associative Property applies to division, check $(a \div b) \div c \stackrel{?}{=} a \div (b \div c)$.

$(27 \div 9) \div 3 \stackrel{?}{=} 27 \div (9 \div 3)$ Pick values for a, b, and c. $6 - 9 = -3$

$(3) \div 3 \stackrel{?}{=} 27 \div (3)$ Simplify. $9 - 6 = 3$

$1 \neq 9$ Simplify.

We found a counterexample. So, division of whole numbers is not associative.

Got It? Do this problem to find out.

1. Is subtraction of decimals associative? If not, give a counterexample.

$(16.5 - 8.6) - 4.3 = 7.9 - 4.3 = 3.6$ $16.5 - (8.6 - 4.3)$ $16.5 - 4.3 = 12.2$

Example 2

Tutor

Name the property shown by each statement.

a. $4 + (a + 3) = (a + 3) + 4$

The order of the numbers and variables changed. This is the Commutative Property of Addition.

b. $1 \cdot (3c) = 3c$

The expression was multiplied by 1 and remained the same. This is the Multiplicative Identity Property.

Got It? Do these problems to find out.

2a. $d + 0 = d$ Additive Identity

2b. $8 \cdot 1 = 8$ Multiplicative Property

2c. $14 + (9 + 10) = (14 + 9) + 10$ Associative Property

2d. $5 \times 7 \times 2 = 7 \times 2 \times 5$ Commutative Property

Simplify Algebraic Expressions

Vocabulary Link
simplify
Everyday Use to make easier to understand
Math Use to shorten an algebraic expression by performing all operations. A simplified expression is often easier to understand.

To **simplify** an algebraic expression, perform all possible operations. You can use the properties you learned in this lesson. Using facts, properties, or rules to reach valid conclusions is called **deductive reasoning**.

Example 3

Tutor

Simplify each expression.

a. $(3 + e) + 7$

$(3 + e) + 7 = (e + 3) + 7$ Commutative Property of Addition
$= e + (3 + 7)$ Associative Property of Addition
$= e + 10$ Simplify.

b. $8 \cdot (x \cdot 5)$

$8 \cdot (x \cdot 5) = 8 \cdot (5 \cdot x)$ Commutative Property of Multiplication
$= (8 \cdot 5) \cdot x$ Associative Property of Multiplication
$= 40x$ Simplify.

c. $9 + (0 + q)$

$9 + (0 + q) = (9 + 0) + q$ Associative Property of Addition
$= 9 + q$ Additive Identity Property

d. $4 \cdot (a \cdot 0)$

$4 \cdot (a \cdot 0) = 4 \cdot 0$ Multiplicative Property of Zero
$= 0$ Multiplicative Property of Zero

3b. $(15+w)+16$ Comm Prop
$(w+15)+16$ Assoc Prop
$w+(15+16)$
$w+31$

3d. $10+(p+18)$
$10+(18+p)$ ← Comm Prop
$(10+18)+p$ ← Assoc. Prop
$28+p$

Got It? Do these problems to find out.

3a. $12 \cdot (10 \cdot z)$

3b. $(15 + w) + 16$

3c. $11 \cdot (b \cdot 0)$

3d. $10 + (p + 18)$

Guided Practice

Check

1. Is subtraction of whole numbers commutative? If not, give a counterexample. (Example 1)

Name the property shown by each statement. (Example 2)

2. $8 \cdot 4 = 4 \cdot 8$

3. $6 \cdot 1 = 6$

4. $9 + 3 + 20 = 3 + 9 + 20$

5. $7 + 0 = 7$

6. $13 + 12 = 12 + 13$

7. $6 \times (1 \times 9) = (6 \times 1) \times 9$

Simplify each expression. (Example 3)

8. $9 + (5 + y)$

9. $3 + (k + 8)$

10. $(15 + s) + 4$

11. $(12 + m) + 4$

12. $(1 \cdot d) \cdot 14$

13. $(z \cdot 11) \cdot 3$

Independent Practice

Go online for Step-by-Step Solutions eHelp

State whether each conjecture is true. If not, give a counterexample. (Example 1)

14. The sum of two odd numbers is always odd.

15. The product of odd numbers is always even.

16. Division of whole numbers is commutative.

17. All multiples of 3 are odd.

Name the property shown by each statement. (Example 2)

18. $0 + 14 = 14$

19. $8 \cdot 1 = 8$

20. $15 + 17 = 17 + 15$

21. $(2 \cdot 8) \cdot 5 = 2 \cdot (8 \cdot 5)$

22. $14 \times 0 \times 3 = 0$

23. $4 + (9 + 2) = (4 + 9) + 2$

24. $7 + x + 11 = x + 7 + 11$

25. $5k \times 1 = 5k$

Simplify each expression. (Example 3)

26. $(d + 12) + 16$

27. $(54 + p) + 16$

28. $14 + (27 + m)$

29. $(r + 32) + 24$

30. $(8 \cdot s) \cdot 9$

31. $g \cdot (5 \cdot 7)$

32. $11 \cdot (t \cdot 4)$

33. $15b(5)$

34. $6(12c)$

35. $(7 + p) + 13$

36. $29 + (1 + t)$

37. $4 \cdot (x \cdot 2)$

38. Use the table to write an expression that shows how many total baskets the Cavaliers made during the season. Simplify the expression.

Cavaliers' Baskets	
free throws	1484
2-point field goals	f
3-point field goals	494

39. Moreno likes to do her social studies homework before she does her math homework. Is doing social studies homework and math homework commutative? Explain your reasoning.

40. CCSS **Justify Conclusions** Ming said, "12, 20, and 36 are all divisible by 2 and by 4. So any whole number that is divisible by 2 is also divisible by 4." Do you agree? If yes, explain your reasoning. If not, give a counterexample.

41. A ceramics class needs to buy blocks of clay for an activity. Use the table at the right to write an expression that represents the total cost of clay, in dollars, for the activity. Simplify the expression.

Ceramics Class	
number of students	19
blocks of clay per student	b
price per block of clay ($)	6

Translate each verbal expression into an algebraic expression. Then simplify the expression.

42. the product of seven and four times a number multiplied by three

43. the sum of two times a number and five added to six times the number

44. eight more than six times a number added to one more than nine times the number

45. the product of eleven and five times a number multiplied by four

46. the difference of twelve times a number and nine times the number

47. the sum of four times a number and seven less than two times the number

Stockbyte /PunchStock

48. Denzel wants to know the total number of boys in the three clubs shown in the table.

Club Membership		
Club	**Boys**	**Girls**
chess	17	19
drama	28	23
music	13	21

a. What expression should Denzel evaluate in order to find the total number of boys?

b. Explain how you can use properties of numbers to make the expression easier to evaluate using mental math.

49. **Financial Literacy** The Center of Wonders science center has the rates shown.

Center of Wonders	
Type of Ticket	**Cost ($)**
admission	a
planetarium	4.50
3-D movie	7.75

a. Write an algebraic expression for the total cost for five people to get into the center, visit the planetarium, and watch a 3-D movie.

b. If the cost of admission to the center is $12, how much will it cost for four people to get into the center and watch a 3-D movie?

c. Children get a discount of $2.50 on their tickets to the planetarium if they also see a 3-D movie. Write an expression to find the cost for two adults and two children to get into the center, see a 3-D movie, and visit the planetarium.

H.O.T. Problems Higher Order Thinking

50. **Identify Structure** Write an algebraic expression that can be simplified using at least two different properties. Simplify the expression showing each step and provide a justification for each step.

51. **Construct an Argument** Is the following statement *true* or *false*? Explain your reasoning.

$$15 + (4 \cdot 6) = (15 + 4) \cdot 6$$

52. **Find the Error** Meghan is simplifying the expression $8(3) \cdot 4 \cdot 2(3)$. Find her mistake and correct it.

$$\begin{aligned} 8(3) \cdot 4 \cdot 2(3) &= 24 \cdot 4 \cdot 2 \\ &= 96 \cdot 2 \\ &= 192 \end{aligned}$$

53. **Persevere with Problems** If you take any two whole numbers and add them together, the sum is always a whole number. This is the Closure Property for Addition. The set of whole numbers is *closed* under addition.

a. Is the set of whole numbers closed under subtraction? If not, give a counterexample.

b. Suppose you had a very small set of numbers that contained only 0 and 1. Would this set be closed under addition? If not, give a counterexample.

c. There is also a Closure Property for Multiplication of Whole Numbers. State this property using the addition property above as a guideline.

d. Is the set {0, 1} closed under multiplication? Explain.

54. **Building on the Essential Question** The number 1 is the identity for multiplication. Do you think that division has an identity? Explain your reasoning.

NASA and The Hubble Heritage Team (STScI/AURA)

Standardized Test Practice

55. Which statement is an example of the Identity Property?

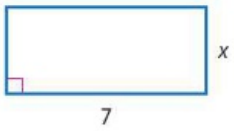
A $3 \cdot x \cdot 0 = 0$

B $7(4x) = (4 \cdot 7)x$

C $5 + (4 + x) = (5 + 4) + x$

D $4x + 0 = 4x$

56. Which expression can be used to find the perimeter of the rectangle below?



F $2x + 7$

G $2x + 14$

H $x + 7$

J $x + 14$

57. Which property is illustrated by the statement below?

$$12 \cdot (n \cdot 5) = (12 \cdot n) \cdot 5$$

A Commutative Property

B Associative Property

C Identity Property

D Zero Property

58. **Short Response** Simplify the expression. Show and justify each step.

$$10 \cdot (x \cdot 3)$$

Common Core Review

Translate each phrase into an algebraic expression. 6.EE.2a

59. Bianca's salary plus a $200 bonus

60. three more than the number of cakes baked

61. six feet shorter than the mountain's height

62. eight less than the quotient of the number of quarters and four

63. STEM The number of times a cricket chirps can be used to estimate the temperature in degrees Fahrenheit. The expression $c \div 4 + 37$, where c is the number of chirps in 1 minute, shows this relationship. 6.EE.2c

a. Find the approximate temperature if a cricket chirps 136 times a minute.

b. What is the temperature if a cricket chirps 100 times in a minute?

Evaluate each expression. 5.OA.1

64. $50 \div 2 \times 5$

65. $6(8 - 4) + 3 \cdot 7$

66. $16 - 2 \cdot 4$

67. $18 + 2 \cdot 3$

68. $49 - 25 + 5$

69. $3(7 \cdot 5) \cdot 2$

70. $90 \div 6 \div 3$

71. $90 \div (6 \div 3)$

72. $20 + 8 \div 4$

Convert each measurement to the given units. 5.MD.1

73. 7 centimeters to meters

74. 72 inches to feet

75. 300 milliliters to liters

76. 24 gallons to quarts

77. 12 kilograms to grams

78. 2 miles to feet

79. 18 yards to feet

80. 9 millimeters to centimeters

ISG Interactive Study Guide
See page 13 for:
• Mid-Chapter Check

COMMUTATIVE PROPERTY ← ORDER

APPLIES TO ADDITION AND MULTIPLICATION

$2+3=5$
$3+2=5$

$2(3)=6$
$3(2)=6$

NOT SUBTRACTION OR DIVISION

$6-4=2$

$4-6=-2$

↑ OPPOSITES

$\frac{8}{2}=4$

$\frac{2}{8}=\frac{1}{4}$

↑ RECIPROCALS

Associative Property ← Grouping

Addition

★ $(2+3)+4 =$
$5 + 4 = 9$

★ $2+(3+4) =$
$2+7 = 9$

Multiplication

★ $(2 \cdot 3) \cdot 4 =$
$6 \cdot 4 = 24$

★ $2 \cdot (3 \cdot 4) =$
$2 \cdot 12 = 24$

$9 \cdot 2 \cdot 5$
$9 \cdot (2 \cdot 5)$
$9 \cdot 10 = 90$

Subtraction

★ $(2-3)-4 =$
$-1 - 4 = -5$

★ $2-(3-4) =$
$2-(-1) = 3$

Division

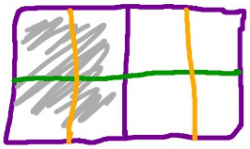
$$\frac{\frac{2}{3}}{4} = \frac{2}{3} \div \frac{4}{1} = \frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

$$\frac{2}{\frac{3}{4}} = \frac{2}{1} \div \frac{3}{4} = \frac{2}{1} \cdot \frac{4}{3} = \frac{8}{3} = 2\frac{2}{3}$$

Multiplicative Identity

$\frac{3}{4} \cdot \boxed{\frac{2}{2}} = \frac{6}{8}$

$\frac{7}{11} \cdot \boxed{\frac{9}{9}} = \frac{63}{99}$



$\frac{1}{2} \cdot \boxed{\frac{2}{2}} = \frac{2}{4}$

$\frac{1}{2} \cdot \boxed{\frac{4}{4}} = \frac{4}{8}$