Lesson 3-4

Dividing Rational Numbers

Interactive Study Guide

See pages 59–60 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

What happens when you add, subtract, multiply, and divide rational numbers?

Common Core State Standards

Content Standards
7.NS.2, 7.NS.2a, 7.NS.2c, 7.NS.3, 7.EE.3

Mathematical Practices
1, 3, 4, 5, 7

Vocabulary

multiplicative inverse
reciprocal

What You'll Learn

- Divide positive and negative fractions using multiplicative inverses.
- Divide algebraic fractions.

Real-World Link

Global Literacy After learning the history of Mexico's holiday *El Día de los Muertos,* or *Day of the Dead,* students created clay containers to commemorate loved ones. They made their containers from two slabs of clay that that they cut into thirds.

Divide Fractions

All of the properties of integers also apply to rational numbers. Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. The statement $\frac{1}{4} \cdot 4 = 1$ demonstrates this property too.

RECIPROCAL

$\frac{4}{5} \cdot \frac{5}{4} = \frac{20}{20} = 1$

Key Concept — Inverse Property of Multiplication

Words	The product of a number and its multiplicative inverse is 1.
Symbols	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Example	$\frac{2}{3} \cdot \frac{3}{2} = 1$

$-\frac{2}{3} \cdot -\frac{3}{2} = +\frac{6}{6} = +1$

Example 1

Tutor

Find the multiplicative inverse of each number.

a. $\frac{7}{16}$

$\frac{7}{16}\left(\frac{16}{7}\right) = 1$ The product is 1.

The multiplicative inverse or reciprocal of $\frac{7}{16}$ is $\frac{16}{7}$.

b. $-6\frac{1}{3}$

$-6\frac{1}{3} = -\frac{19}{3}$ Write $-6\frac{1}{3}$ as an improper fraction.

$-\frac{19}{3}\left(-\frac{3}{19}\right) = 1$ The product is 1.

The multiplicative inverse or reciprocal of $-6\frac{1}{3}$ is $-\frac{3}{19}$.

Got It? Do these problems to find out.

1a. $-\frac{7}{9}$

1b. $2\frac{1}{12}$

Key Concept Divide Fractions

Words To divide by a fraction, multiply by its multiplicative inverse.

Examples $\frac{4}{9} \div \frac{3}{5} = \frac{4}{9} \cdot \frac{5}{3}$ $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where b, c, and $d \neq 0$

To demonstrate this concept, consider $\frac{4}{9} \div \frac{3}{5}$ and $\frac{a}{b} \div \frac{c}{d}$.

$$\frac{\frac{4}{9}}{\frac{3}{5}} = \frac{\frac{4}{9} \cdot \frac{5}{3}}{\frac{3}{5} \cdot \frac{5}{3}}$$ Multiply the numerator and denominator by $\frac{5}{3}$, the multiplicative inverse of $\frac{3}{5}$.

$$= \frac{\frac{4}{9} \cdot \frac{5}{3}}{1}$$ $\frac{3}{5} \cdot \frac{5}{3} = 1$

$$= \frac{4}{9} \cdot \frac{5}{3}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}}$$ Multiply the numerator and denominator by $\frac{d}{c}$, the multiplicative inverse of $\frac{c}{d}$.

$$= \frac{\frac{a}{b} \cdot \frac{d}{c}}{1}$$ $\frac{c}{d} \cdot \frac{d}{c} = 1$

$$= \frac{a}{b} \cdot \frac{d}{c}$$

Dividing By a Whole Number
When dividing by a whole number, always rename it as an improper fraction first. Then multiply by its reciprocal.

Example 2

Find each quotient. Write in simplest form.

a. $\frac{1}{9} \div \frac{5}{12}$

$$\frac{1}{9} \div \frac{5}{12} = \frac{1}{9} \cdot \frac{12}{5}$$ Multiply by the reciprocal of $\frac{5}{12}$, $\frac{12}{5}$.

$$= \frac{1}{\cancel{9}_{3}} \cdot \frac{\cancel{12}^{4}}{5}$$ Divide by the GCF, 3.

$$= \frac{4}{15}$$ Simplify.

b. $\frac{3}{7} \div 8$

$$\frac{3}{7} \div 8 = \frac{3}{7} \div \frac{8}{1}$$ Write 8 as $\frac{8}{1}$.

$$= \frac{3}{7} \cdot \frac{1}{8}$$ Multiply by the reciprocal of $\frac{8}{1}$, $\frac{1}{8}$.

$$= \frac{3}{56}$$ Simplify.

Got It? Do these problems to find out.

2a. $\frac{1}{3} \div \frac{7}{15}$ **2b.** $\frac{5}{8} \div \left(-\frac{3}{4}\right)$ **2c.** $\frac{3}{4} \div 11$ **2d.** $-\frac{6}{7} \div 12$

Example 3

Find $-4\frac{2}{3} \div 3\frac{1}{9}$.

$$-4\frac{2}{3} \div 3\frac{1}{9} = -\frac{14}{3} \div \frac{28}{9}$$ Rename the mixed numbers as improper fractions.

$$= -\frac{14}{3} \cdot \frac{9}{28}$$ Multiply by the reciprocal, $\frac{9}{28}$.

$$= -\frac{\cancel{14}^{1}}{\cancel{3}_{1}} \cdot \frac{\cancel{9}^{3}}{\cancel{28}_{2}}$$ Divide out common factors.

$$= -\frac{3}{2} \text{ or } -1\frac{1}{2}$$ Simplify.

Got It? Do these problems to find out.

3a. Find $6\frac{3}{8} \div \left(-4\frac{1}{4}\right)$. **3b.** Find $-6\frac{4}{5} \div \left(-2\frac{2}{5}\right)$.

Division can be used to find the number of equal size groups in a real-world situation.

Example 4

Tutor

Tessa feeds her dog Roscoe $3\frac{3}{4}$ cups of dog food per day. If she buys a bag of food that contains 165 cups, how many days will the bag of food last?

To find how many days, divide. $165 \div 3\frac{3}{4}$ THINK How many $3\frac{3}{4}$s are in 165?

$165 \div 3\frac{3}{4} = \frac{165}{1} \div \frac{15}{4}$ Rewrite 165 and $3\frac{3}{4}$ as improper fractions.

$= \frac{165}{1} \cdot \frac{4}{15}$ Multiply by the reciprocal of $\frac{15}{4}$, $\frac{4}{15}$.

$= \frac{\overset{11}{\cancel{165}}}{1} \cdot \frac{4}{\underset{1}{\cancel{15}}}$ Divide out common factors.

$= 44$ Simplify.

So, the bag of dog food will last 44 days.

Check

Tessa feeds her dog about 4 cups of food for a little longer than 40 days, so the number of cups the bag contains should be about 4(40) or 160 cups. The answer is reasonable.

Watch Out!

When working with fractions, dividing just the whole number portions may not give a good estimate for an answer. First round each fraction or mixed number to the nearest whole number, then divide.

Got It? Do this problem to find out.

4. A box of cereal contains $15\frac{3}{5}$ ounces. If one bowl holds $2\frac{2}{5}$ ounces of cereal, how many bowls of cereal are in one box?

Divide Algebraic Expressions

You can divide algebraic fractions in the same way that you divide numerical fractions.

Example 5

Tutor

Find $\frac{5}{3ab} \div \frac{15}{abc}$. Write the quotient in simplest form.

$\frac{5}{3ab} \div \frac{15}{abc} = \frac{5}{3ab} \cdot \frac{abc}{15}$ Multiply by the reciprocal of $\frac{15}{abc}$, $\frac{abc}{15}$.

$= \frac{\overset{1}{\cancel{5}}}{3\cancel{ab}_{1}} \cdot \frac{\overset{1}{\cancel{abc}}}{\underset{3}{\cancel{15}}}$ Divide out common factors.

$= \frac{c}{9}$ Simplify.

Got It? Do these problems to find out.

Find each quotient. Write in simplest form.

5a. $\frac{5ab}{6} \div \frac{10b}{7}$

5b. $\frac{mn}{4} \div \frac{m}{8}$

Guided Practice

Find the multiplicative inverse of each number. (Example 1)

1. $\frac{6}{7}$
2. $-5\frac{1}{2}$
3. -63
4. $9\frac{9}{10}$
5. -101
6. $\frac{11}{35}$

Find each quotient. Write in simplest form. (Examples 2 and 3)

7. $-\frac{4}{5} \div \frac{8}{9}$
8. $-\frac{5}{7} \div \frac{2}{35}$
9. $\frac{4}{9} \div (-2)$
10. $\frac{7}{9} \div (-14)$
11. $-2\frac{1}{5} \div \left(-3\frac{2}{3}\right)$
12. $7\frac{1}{9} \div \left(-1\frac{1}{3}\right)$

13. Sonia is making a quilted wall hanging that is 38 inches wide. If each quilt square is $4\frac{3}{4}$ inches wide, how many squares will she need to complete one row of the wall hanging? (Example 4)

Find each quotient. Write in simplest form. (Example 5)

14. $\frac{4ab}{c} \div \frac{3a}{2c}$
15. $\frac{mn}{6} \div \frac{3m}{p}$
16. $\frac{3xy}{yz} \div \frac{6y}{5}$

Independent Practice

Go online for Step-by-Step Solutions

Find the multiplicative inverse of each number. (Example 1)

17. $-\frac{4}{5}$
18. $6\frac{1}{8}$
19. $\frac{10}{19}$
20. $-4\frac{2}{7}$
21. 19
22. -54
23. $5\frac{2}{3}$
24. -12
25. $-\frac{2}{9}$

Find each quotient. Write in simplest form. (Examples 2 and 3)

26. $-\frac{1}{8} \div \frac{2}{5}$
27. $-\frac{5}{12} \div \frac{2}{3}$
28. $-\frac{6}{7} \div \left(-\frac{16}{21}\right)$
29. $-\frac{4}{9} \div (-24)$
30. $-\frac{9}{10} \div (-21)$
31. $-6\frac{1}{9} \div 3\frac{2}{3}$
32. $-10\frac{3}{5} \div \left(-2\frac{2}{5}\right)$
33. $2\frac{3}{8} \div 1\frac{1}{6}$

34. Hannah is making chocolate chip cookies. The dry ingredients are shown at the right. How many batches of cookies can she make if she has $7\frac{1}{2}$ cups of brown sugar? (Example 4)

Chocolate Chip Cookies
$\frac{1}{2}$ cup granulated sugar
$1\frac{1}{2}$ cups packed brown sugar
$2\frac{1}{2}$ cups all-purpose flour
$\frac{3}{4}$ teaspoon salt
1 teaspoon baking powder
1 teaspoon baking soda

35. How many play costumes can be made with $49\frac{1}{2}$ yards of fabric if each costume requires $4\frac{1}{8}$ yards? (Example 4)

12 COSTUMES

Find each quotient. Write in simplest form. (Example 5)

36. $\frac{x}{20} \div \frac{x}{5}$
37. $\frac{m}{6n} \div \frac{7m}{3n}$
38. $\frac{m}{np} \div \frac{3m}{2p}$
39. $\frac{5a}{3bc} \div \frac{2a}{9bc}$

(31)

$$\frac{m}{6n} \div \frac{7m}{3n}$$

$$\frac{m}{6n} \cdot \frac{3n}{7m} = \frac{m \cdot 3n}{6n \cdot 7m} = \frac{3mn}{42mn} = \frac{1}{14}$$

$$\frac{3 \cdot 1 \cdot m \cdot n}{3 \cdot 14 \cdot m \cdot n}$$

$12 \div 3 = 4$

12 CAN BE EQUALLY DIVIDEDED IN 4 GROUPS OF 3

$12 \cdot \frac{1}{3} = 4$

WHAT IS $\frac{1}{3}$ OF 12

$60\left(\frac{1}{5}\right) = 60 \div \frac{5}{1} = 12$

$\frac{1}{4} \div \frac{3}{5} \div \frac{2}{10}$

AARON

DEVIN

$\frac{1}{4} \div \frac{3}{5}$

$\frac{1}{4} \cdot \frac{5}{3} = \frac{5}{12}$

$\frac{5}{12} \div \frac{2}{10} = \frac{5}{12} \cdot \frac{\cancel{10}^{5}}{2} = \frac{25}{12} = 2\frac{1}{12}$

$\frac{1}{4} \div \left(\frac{3}{5} \div \frac{2}{10}\right)$

40. Barbara babysat for $3\frac{1}{4}$ hours and earned \$19.50. What was her hourly rate?

41 A train traveled 405 miles in $4\frac{1}{2}$ hours. How fast was the train traveling on average? (*Hint:* Distance equals the rate multiplied by the time.)

42. Sydney reduced her favorite photograph to put in a scrapbook. How many times as wide is the actual photo than the reduced photo?

4 in.

3 in.

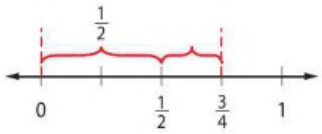
Evaluate each expression if $m = 2\frac{2}{5}$, $n = -\frac{3}{10}$, and $p = 6$.

43. $mn \div p$ **44.** $\frac{m}{n}$ **45.** $np \div m$

46. Ms. Augello is making tie-dyed shirts with her students. Each gallon of hot water needs $\frac{2}{3}$ cup of dye. If Ms. Augello has $5\frac{1}{4}$ cups of dye, how many batches of solution will she be able to make?

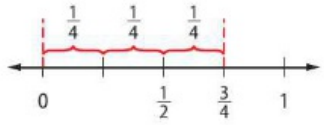
47. CCSS **Make a Conjecture** The model at the left shows $\frac{3}{4} \div \frac{1}{2}$. The model at the right shows $\frac{3}{4} \div \frac{1}{4}$.

How many $\frac{1}{2}$s are in $\frac{3}{4}$?



There are $1\frac{1}{2}$ $\frac{1}{2}$s in $\frac{3}{4}$.

How many $\frac{1}{4}$s are in $\frac{3}{4}$?



There are three $\frac{1}{4}$s in $\frac{3}{4}$.

Make a conjecture about what happens to the quotient as the value of the divisor increases. Test your conjecture.

H.O.T. Problems Higher Order Thinking

48. CCSS **Use Math Tools** Choose two fractions and use an area model or number line to show that division of rational numbers is not commutative.

49. CCSS **Persevere with Problems** Give a counterexample to this statement. *The quotient of two fractions between 0 and 1 is always a whole number.*

50. CCSS **Construct an Argument** Which is greater, $40 \cdot \frac{1}{4}$ or $40 \div \frac{1}{4}$? Explain.

51. CCSS **Identify Structure** Is a whole number divided by a proper fraction *always*, *sometimes*, or *never* greater than the whole number?

52. **Building on the Essential Question** Explain why, for a positive number n, $n \div \frac{1}{2} > n$.

Standardized Test Practice

53. Heidi is having a party. She is planning that each of her 16 guests will have $\frac{3}{4}$ cup of snack mix. She has made 12 cups of snack mix. Which expression could Heidi use to determine if she has made enough snack mix for each of her guests?

A $16 \div \frac{3}{4}$ **C** $12 \div \frac{3}{4}$

B $16 \div 12$ **D** $\frac{3}{4}(12)$

54. A bag of potting soil contains $4\frac{1}{4}$ pounds of soil. Each flower that Mr. Henderson plants will need $\frac{1}{8}$ pound of soil. How many flowers will he be able to plant?

F 16 **H** 32

G 28 **J** 34

55. A recipe for one batch of soft pretzels calls for $\frac{1}{4}$ cup of salt and $\frac{2}{3}$ cup of sugar. If Mrs. Valdez uses $\frac{7}{8}$ cup of salt and $2\frac{1}{3}$ cups of sugar, how many batches of pretzels is she making?

A $3\frac{1}{2}$ **C** $2\frac{1}{4}$

B 3 **D** 2

56. **Short Response** Popcorn is sold in a variety of sizes. Use the table to find how many times as large the regular bag of popcorn is than the snack bag.

Size	Amount (cups)
Snack	$3\frac{1}{2}$
Regular	$8\frac{3}{4}$
Large	12

Common Core Review

Find each product. Write in simplest form 7.NS.2a

57. $2 \cdot \frac{9}{16}$ **58.** $-4\frac{4}{7} \cdot 2\frac{5}{8}$ **59.** $\frac{3}{20} \cdot \left(-\frac{10}{11}\right)$

60. $-6\frac{1}{2} \cdot \left(-3\frac{1}{4}\right)$ **61.** $-\frac{5}{6} \cdot \left(-1\frac{7}{35}\right)$ **62.** $1\frac{1}{8} \cdot 1\frac{1}{3}$

63. The White House covers an area of 0.028 square mile. What fraction of a square mile is this? 8.NS.1

64. The Wildcat football team was penalized the same amount four times during the third quarter. The total of the four penalties was 60 yards. If -60 represents a loss of 60 yards, write a division sentence to represent this situation. Then express the number of yards of each penalty as an integer. 7.EE.3

Find each product. 7.NS.2

65. $12(-6)$ **66.** $-12(-11)$ **67.** $4(-2)(-6)$

Find each sum or difference. 7.NS.1

68. $23 - (-13)$ **69.** $-42 + (-26)$ **70.** $-80 - (-80)$

71. $n + 2n$ **72.** $-4x - (-3x)$ **73.** $5n - 10n$

Translate each of the following to a mathematical expression. 6.EE.2a

74. Add 7 and 5, and then multiply the result by 3.

75. Subtract the quotient of 6 and 3 from 12.

76. Divide the sum of 10 and 15 by 5.

77. Subtract the sum of 3 and 5 from 15.

$2\frac{3}{8} \div 1\frac{1}{6}$

$2 \times 8 + 3 = 19$

$1 \times 6 + 1 = 7$

$19 \times 3 = 57$

① $\frac{19}{8} \div \frac{7}{6}$

② $\frac{19}{8} \cdot \frac{6}{7} = \frac{57}{28} = 2\frac{1}{28}$

$\frac{28}{28} + \frac{28}{28} + \frac{1}{28}$

$$-6\frac{1}{9} \div 3\frac{2}{3}$$

① $\frac{55}{9} \div \frac{11}{3}$

② $\frac{\overset{5}{\cancel{55}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{11}}} = \left(-\frac{5}{3}\right) = -1\frac{2}{3}$