Lesson 4-1
Powers and Exponents

Interactive Study Guide

See pages 73–74 for:
- Getting Started
- Vocabulary Start-Up
- Notes

Essential Question

Why is it useful to write numbers in different ways?

Common Core State Standards

Content Standards
8.EE.1

Mathematical Practices
1, 3, 4, 6, 8

Vocabulary

exponent

power

base

What You'll Learn

- Write expressions using exponents.
- Evaluate expressions containing exponents.

Real-World Link

Computers Data storage capacity is measured in bytes and is based on powers of 2. The standard scientific meanings for the prefixes *mega-* and *giga-* are one million and one billion, respectively. In computer science, a megabyte equals 2^{20} bytes and a gigabyte equals 2^{30} bytes.

Use Exponents

An expression like $5 \cdot 5 \cdot 5$ with equal factors can be written using an exponent. An **exponent** tells how many times a number is used as a factor. A number that is expressed using an exponent is called a **power**. The number that is multiplied is called the **base**. So, $5 \cdot 5 \cdot 5$ equals the power 5^3.

base ⋯➤ 5^3 ◀⋯ exponent

power

Read and Write Powers

Power	Words	Factors
5^1	5 to the first power	5
5^2	5 to the second power or 5 squared	$5 \cdot 5$
5^3	5 to the third power or 5 cubed	$5 \cdot 5 \cdot 5$
5^4	5 to the fourth power or 5 to the fourth	$5 \cdot 5 \cdot 5 \cdot 5$
⋮	⋮	⋮
5^n	5 to the nth power or 5 to the nth	$\underbrace{5 \cdot 5 \cdot 5 \cdot \ldots \cdot 5}_{n \text{ factors}}$

$x^2 = x$ SQUARED $y^3 = y$ CUBED

Example 1

Tutor

Write each expression using exponents.

a. $(-8) \cdot (-8) \cdot (-8)$

The base -8 is a factor 3 times.

$(-8) \cdot (-8) \cdot (-8) = (-8)^3$

b. $(k + 2)(k + 2)(k + 2)(k + 2)$

The base $(k + 2)$ is a factor 4 times.

$(k + 2)(k + 2)(k + 2)(k + 2) = (k + 2)^4$

c. $5 \cdot r \cdot r \cdot s \cdot s \cdot s \cdot s$

$5 \cdot r \cdot r \cdot s \cdot s \cdot s \cdot s = 5 \cdot (r \cdot r) \cdot (s \cdot s \cdot s \cdot s)$ Group factors with like bases.

$= 5 \cdot r^2 \cdot s^4$ or $5r^2s^4$ $r \cdot r = r^2, s \cdot s \cdot s \cdot s = s^4$

EXPONENT ↓ 2

BASE ➤ 3 ← POWER

Got It? Do these problems to find out.

1a. $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

1b. $x \cdot x \cdot x \cdot x \cdot x$

1c. $(c - d)(c - d)$

1d. $9 \cdot f \cdot f \cdot f \cdot f \cdot g$

Vocabulary Link
Evaluate
Everyday Use Determine the significance or worth of something.
Math Use Find the value of an expression.

Evaluate Expressions

Since powers represent repeated multiplication, they need to be included in the rules for order of operations.

Concept Summary Order of Operations

Step 1	Simplify the expressions inside grouping symbols.
Step 2	Evaluate all powers.
Step 3	Multiply and/or divide in order from left to right.
Step 4	Add and/or subtract in order from left to right.

Example 2

Tutor

Exponents
An exponent goes with the number, variable, or quantity in parentheses immediately preceding it.

The playing area for beach volleyball includes the playing court and the free zone. Evaluate each expression to find the area of the playing court and the free zone.

a. The playing court is a rectangle with an area of 2^7 square meters.

$2^7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ 2 is a factor 7 times.
$= 128$ Simplify.

The area of the playing court is 128 square meters.

b. The area of the free zone is $2^2 \cdot 3^2 \cdot 5$ square meters.

$2^2 \cdot 3^2 \cdot 5 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$ Evaluate powers.
$= 180$ Multiply.

The area of the free zone is 180 square meters.

Got It? Do this problem to find out.

2. STEM A tennis ball is dropped from the top of a building. After 8 seconds, the tennis ball hits the ground. The distance in meters the ball traveled is represented by $4.9(8)^2$. How far did the ball drop?

Watch Out!
$(-3)^2$ is not the same as -3^2.
- $(-3)^2 = (-3)(-3)$
  $= 9$
- $-3^2 = (-1)(3^2)$
  $= -9$

Example 3

Tutor

Evaluate $x^2 + y^3$ if $x = 6$ and $y = -2$.

$x^2 + y^3 = 6^2 + (-2)^3$ Replace x with 6 and y with -2.
$= 36 - 8$ Evaluate powers; $6^2 = (6 \cdot 6)$ or 36; $(-2)^3 = (-2)(-2)(-2)$ or -8.
$= 28$ Subtract.

Got It? Do these problems to find out.

Evaluate each expression if $a = 5$, $b = -2$, and $c = \frac{3}{4}$.

3a. $10 + b^2$ **3b.** $(a + b)^3$ **3c.** $2 - c^2$

Guided Practice

Write each expression using exponents. (Example 1)

1. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
2. $d \cdot d \cdot d \cdot d \cdot d \cdot d$
3. $\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)$
4. $4 \cdot m \cdot m \cdot m \cdot q \cdot q \cdot q$
5. $(y-3)(y-3)(y-3)$
6. $(a+1)(a+1)$

7. The longhorn beetle can have a body length of more than 2^4 centimeters. How many centimeters long is this? (Example 2)

8. STEM Theo sends an E-mail to three friends. Each friend forwards the E-mail to three friends. Each of those friends forwards it to three friends, and so on. Write the number of E-mails sent during the fifth stage as a power. Then find the value of the power. (Example 2)

Evaluate each expression if $a = 3$, $b = -4$, and $c = 3.5$. (Example 3)

9. $a^3 + 2$
10. $3(b-1)^2$
11. $c^2 + b^2$
12. $4c - 7 + b^3$

Independent Practice

Go online for Step-by-Step Solutions

Write each expression using exponents. (Example 1) n^2 = n SQUARED x^3 = x CUBED

13. $11 \cdot 11 \cdot 11 \cdot 11$
14. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
15. $(-8)(-8)(-8)(-8)(-8)(-8)$
16. $(-14) \cdot (-14) \cdot (-14)$
17. $\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)$
18. $(-1.5)(-1.5)(-1.5)$
19. $ab \cdot ab \cdot ab \cdot ab$
20. $5 \cdot p \cdot p \cdot p \cdot q \cdot q \cdot q$
21. $3 \cdot 7 \cdot m \cdot m \cdot n \cdot n \cdot n \cdot n = 21m^2n^4$
22. $8(c+4)(c+4)$
23. $(n-5)(n-5)(n-5)$
24. $(2x+3y)(2x+3y)$

BASE → 3^4 ← EXPONENT = $3 \cdot 3 \cdot 3 \cdot 3$

POWER

THREE AS A FACTOR FOUR TIMES

25. STEM The longest chain of active volcanoes is in the South Pacific. This chain is more than $3 \cdot 10^4$ miles long and has approximately $3^5 \cdot 5$ volcanoes. (Example 2)
    a. How long is the chain of volcanoes?
    b. How many volcanoes are there?

$(7x)^2 = 7 \cdot x \cdot 7 \cdot x$

26. A water park has a wave pool that contains about $2^6 \cdot 4^3 \cdot 10^2$ gallons of water. How many gallons of water is this? (Example 2)

$3^4 = 81$

$3 \cdot 3 = 9$ $3 \cdot 3 = 9$ $9 \cdot 9 = 81$

$3^4 = 12$

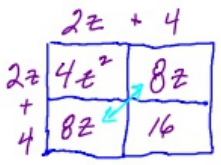
Evaluate each expression if $x = -2$, $y = 3$, and $z = 2.5$. (Example 3)

$7(4) = 28$

$7(-2)(-2)$

27. y^4
28. z^3
29. $7x^2$
30. xy^3 $x \cdot y \cdot y \cdot y$
31. $z^2 + x$
32. $y^4 + 9$
33. $2y + z^3$
34. $x^2 + 2y - 3$
35. $y^2 - 3x + 8$
36. $4(y+1)^4$
37. $3(2z+4)^2$ $3(2z+4)(2z+4)$
38. $5(x^3 + 6)$

$3(2z+4)^2$



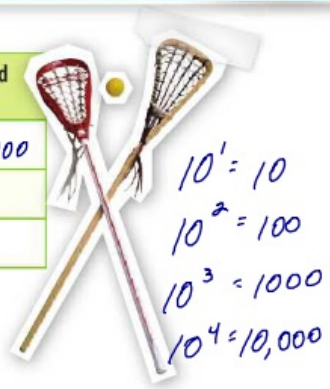
$4 \cdot 2 \cdot z = 8z$

$3(4z^2+16z+16)$

$12z^2+48z+48$

39 CCSS **Be Precise** The table shows the minimum areas of different sports fields.

Sport	Minimum Field Area (ft^2)
field hockey	$2^6 \cdot 10^3$
men's lacrosse	$3^2 \cdot 7 \cdot 10^3$
women's soccer	$2^4 \cdot 5^2 \cdot 7 \cdot 13$



a. Find the minimum area of each playing field.

b. Order the areas from least to greatest.

c. How much greater is the area of a field hockey field than the area of a men's lacrosse field?

Evaluate each expression.

40. 9^2

41. 11^3

42. $\left(-\frac{2}{3}\right)^3$

43. $(-5)^4$

44. $(-2)^7$

45. $2 \cdot 4^4$

46. $6^3 \cdot 4$

47. $3^5 \cdot 10$

48. $2^2 \cdot 10$

49. $7^3 \cdot 2^2$

50. $5 \cdot 2^4$

51. $(4.5)^4 \cdot 2$

Replace each ● with <, >, or = to make a true statement.

52. 2^5 ● 5^2

53. 3^6 ● 6^3

54. 2^6 ● 8^2

55. 8^3 ● 4^5

56. $(-6)^4$ ● 6^4

57. $(-4)^6$ ● $(-4)^7$

58. CCSS **Multiple Representations** In this problem, you will explore volume of a cube. The volume of a cube equals the side length cubed.

a. Symbols Write an equation showing the relationship between side length s and volume V of a cube.

b. Table Make a table of values showing the volume of a cube with side lengths of 1, 2, 4, 8, and 16 centimeters.

c. Analyze Use your table to make a conjecture about the change in volume when the side length of a cube is doubled. Justify your response by writing an algebraic expression.

H.O.T. Problems Higher Order Thinking

59. CCSS **Model with Mathematics** Write a real-world problem that involves multiplying two expressions with exponents. Then solve.

60. CCSS **Persevere with Problems** Determine whether x^3 is *always*, *sometimes*, or *never* a positive number for $x \neq 0$. Explain your reasoning.

61. CCSS **Justify Conclusions** Suppose the population of the United States is about 230 million. Is this number closer to 10^7 or 10^8? Explain.

62. CCSS **Identify Repeated Reasoning** Use the pattern below to predict the value of 5^0. Explain your reasoning.

$$5^4 = 625$$
$$5^3 = 125$$
$$5^2 = 25$$
$$5^1 = 5$$

63. **Building on the Essential Question** Describe the advantages of using exponents to represent numeric values.

Standardized Test Practice

64. Marta observed that a bacterium cell doubled every 3 minutes.

Time (min)	Number of Bacteria
3	2^1
6	2^2
9	2^3
12	2^4

Which expression represents the number of cells after one-half hour?

A 2^{10} **C** 2^{20}

B 2^{15} **D** 2^{30}

65. Short Response Suppose a certain forest fire doubles in size every 8 hours. If the initial size of the fire was 1 acre, how many acres will the fire cover in 3 days?

66. Which of the following is equivalent to $4^3 \cdot 5^2$?

F $12 \cdot 25$

G $3 \cdot 3 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

H $4 \cdot 4 \cdot 4 \cdot 5 \cdot 5$

J $4 \cdot 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5$

67. Evaluate $\left(\frac{4}{5}\right)^2$.

A $\frac{8}{25}$ **C** $\frac{8}{10}$

B $\frac{16}{25}$ **D** $1\frac{3}{5}$

Common Core Review

Find each sum or difference. 7.NS.1

68. $-12 + (-7)$ **69.** $25 - (-5)$ **70.** $-15 + 8$

71. $-9 - (-9)$ **72.** $3 + (-11)$ **73.** $-18 - 2$

Name the property shown by each statement. 7.EE.1

74. $87 + 0 = 0$ **75.** $19 \times 5 = 5 \times 19$ **76.** $12 \cdot 0 = 0$

77. Kari grew $1\frac{5}{8}$ inches last year and $2\frac{3}{4}$ inches this year. How many total inches did Kari grow in the past two years? 7.NS.3

78. A dance instructor charges a sign-up fee of \$50 plus \$8 for each group lesson. Write an expression that can be used to find the total cost of dance lessons. Then find the cost of 15 lessons. 7.EE.4

Write an integer for each situation. Then identify its opposite and explain its meaning. 7.NS.1a

79. 150 feet below sea level

80. a profit of \$75

Find the greatest common factor for each pair of numbers. 6.NS.4

81. 8 and 12 **82.** 18 and 24 **83.** 12 and 14

84. 27 and 36 **85.** 57 and 63 **86.** 45 and 80

Find each quotient. 7.NS.2b

87. $-24 \div (-6)$ **88.** $60 \div (-4)$ **89.** $-56 \div 8$ **90.** $-81 \div (-3)$