Lesson 3-1

Fractions and Decimals

Ⓔ Essential Question

What happens when you add, subtract, multiply, and divide rational numbers?

CCSS Common Core State Standards

Content Standards
7.NS.2, 7.NS.2d, 8.NS.1, 7.EE.3

Mathematical Practices
1, 3, 4, 5, 7

Vocabulary

repeating decimal
terminating decimal
bar notation

What You'll Learn

• Write fractions as terminating or repeating decimals.
• Compare fractions and decimals.

Real-World Link

Robotics If you could design your own robot, what would it look like? What would it be able to do? In an annual robot competition, middle school students apply math and science to design, program, and test their own robots. The goal is to make their 'bots outperform the competition!
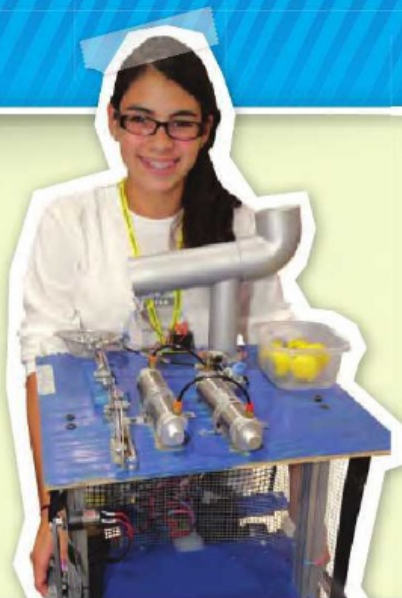
Write Fractions as Decimals

Some fractions like $\frac{1}{2}$ and $\frac{3}{4}$ can be written as a decimal by making equivalent fractions with denominators of 10, 100, or 1000. However, any fraction $\frac{a}{b}$, where $b \neq 0$, can be written as a decimal by dividing the numerator by the denominator. So, $\frac{a}{b} = a \div b$. The decimal form of a rational number is called a **repeating decimal**.

If the repeating digit is zero, then the decimal is a **terminating decimal**.

Example 1

Write $\frac{7}{8}$ as a decimal.

Method 1 Use paper and pencil.

$$\begin{array}{r} 0.875 \\ 8{\overline{)7.000}} \\ -6\ 4 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Place the decimal point. Annex zeros and divide as with whole numbers.

Division ends when the repeating digit is 0.

Method 2 Use a calculator.

7 ÷ 8 ENTER 0.875

Using either method, $\frac{7}{8} = 0.875$.

Got It? Do these problems to find out.

Write each fraction as a decimal.

1a. $\frac{4}{5} = \frac{8}{10} = 0.8$

1b. $\frac{3}{16}$

Handwritten notes:

$5\overline{)4}$

$\frac{4}{5} = \frac{8}{10} = 0.8$ (×2)

$2^1 \rightarrow \frac{1}{2} = 1$ PLACE 0.5

$2^2 \rightarrow \frac{1}{4}$ OR $\frac{3}{4} = 2$ PLACE

$2^3 \rightarrow \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8} = 3$ PLACES

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Vocabulary Link

Terminating

Everyday Use bringing to an end

Math Use a decimal whose digits end

Not all fractions have repeating digits that are zero. Sometimes a nonzero digit or a group of digits repeats without end in the quotient

$\frac{1}{6} \longrightarrow$

```
    0.166
 6)1.000
   −6
    40
   −36
    40
   −36
     4
```

The digit 6 repeats.

The remainder after each step is 4.

Check 1 ÷ 6 ENTER 0.1666666667 ✓ The last digit is rounded.

You can indicate that the digit 6 repeats by annexing dots. So,
$\frac{1}{6} = 0.1666666666\ldots$. This decimal is called a repeating decimal.

Repeating decimals have a pattern in their digits that repeats without end. **Bar notation** is a bar or line placed over the digit(s) that repeats. The table shows some examples of repeating decimals and their bar notations.

Decimal	Bar Notation
0.166666…	$0.1\overline{6}$
0.353535…	$0.\overline{35}$
12.6888888…	$12.6\overline{8}$
5.714285714285…	$5.\overline{714285}$

Example 2

Tutor

Write each fraction as a decimal. Use a bar to show a repeating decimal.

a. $\frac{5}{12}$

$\frac{5}{12} \rightarrow$
```
   0.4166...
12)5.0000...
```
The digit 6 repeats.

So, $\frac{5}{12} = 0.41\overline{6}$.

b. $-\frac{2}{11}$

$-\frac{2}{11} \rightarrow$
```
   0.1818...
11)2.0000...
```
The digits 18 repeat.

So, $-\frac{2}{11} = -0.\overline{18}$.

Got It? Do these problems to find out.

2a. $-\frac{5}{6}$

2b. $\frac{7}{9}$

It is helpful to memorize these fraction-decimal equivalents.

MEMORIZE THESE

PLUS THE 1/9 THS

$\frac{1}{9} = 0.\overline{1}$

$\frac{2}{9} = 0.\overline{2}$ *ECT.*

Concept Summary	Fraction-Decimal Equivalents				
$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.\overline{3}$	$\frac{1}{4} = 0.25$	$\frac{1}{5} = 0.2$	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$
$\frac{2}{3} = 0.\overline{6}$	$\frac{3}{4} = 0.75$	$\frac{2}{5} = 0.4$	$\frac{3}{5} = 0.6$	$\frac{4}{5} = 0.8$	$\frac{5}{6} = 0.8\overline{3}$

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Example 3

According to the USDA, teenage boys should consume an average of 2700 Calories per day. About 360 Calories should come from milk. To the nearest hundredth, what part of a teenage boy's total Calories should come from milk?

Divide the number of Calories that should come from milk, 360, by the number of total Calories, 2700.

$$360 \boxed{\div} 2700 \boxed{\text{ENTER}} \ 0.133... \text{ or } 0.1\overline{3}$$

Look at the digit to the right of the thousandths place. Round down since $3 < 5$.

Milk should be 0.13 of the daily Calories consumed by a teenage boy.

Got It? Do this problem to find out.

3. In a recent Masters Tournament, Zach Johnson's first shot landed on the fairway 45 out of 56 times. To the nearest thousandth, what part of the time did his shot land on the fairway?

Compare Fractions and Decimals

It may be easier to compare numbers when they are written as decimals.

Example 4

Comparing Decimals

When comparing two decimals, compare the digits in the same place-value position.

Replace each ● with $<$, $>$, or $=$ to make a true sentence.

a. $\frac{1}{4}$ ● 0.2

$\frac{1}{4}$ ● 0.2   Write the sentence.

0.25 ● 0.2**0**   Write $\frac{1}{4}$ as a decimal. Annex a zero to 0.2.
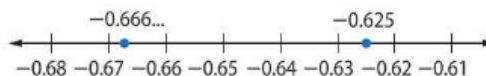
0.25 > 0.20   In the hundredths place, $5 > 0$.

Check Since 0.20 is to the left of 0.25 on the number line, $\frac{1}{4} > 0.2$.

b. $-\frac{5}{8}$ ● $-\frac{6}{9}$

Write the fractions as decimals and then compare the decimals.

$$-\frac{5}{8} = -0.625 \qquad -\frac{6}{9} = -0.666... \text{ or } -0.\overline{6}$$

Since -0.625 is to the right of $-0.\overline{6}$ on the number line, $-\frac{5}{8} > -\frac{6}{9}$.

Got It? Do these problems to find out.

4a. $\frac{7}{8}$ ● 0.87   4b. $-\frac{7}{15}$ ● $-\frac{5}{12}$

Tools **Tutor**

### Example 5

Real World

Use a Graph
You can use a graph to visualize data, analyze trends, and make predictions. In this example, you can compare the decimals on a number line.

Thirty out of 36 seventh graders and 34 out of 40 eighth graders participated in a marathon for charity. Which class had a greater fraction participating in the marathon?

Write each fraction as a decimal.
Then compare the decimals.

seventh graders: $\frac{30}{36} = 0.8\overline{3}$

eighth graders: $\frac{34}{40} = 0.85$

$$0.8\overline{3} \qquad 0.85$$

0.80 0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88
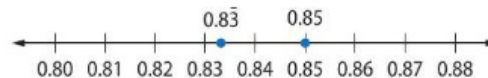
On a number line, $0.8\overline{3}$ is to the left of 0.85. Since $0.8\overline{3} < 0.85$, $\frac{30}{36} < \frac{34}{40}$.

So, a greater fraction of eighth graders participated in the marathon.

Got It? Do this problem to find out.

5. Over the weekend, $\frac{16}{28}$ of the eighth grade girls and $\frac{19}{30}$ of the eighth grade boys went to see a new comedy movie. Did a greater fraction of girls or boys see the movie?

Guided Practice

Check ✓

↓ BAR NOTATION

Write each fraction as a decimal. Use a bar to show a repeating decimal. (Examples 1 and 2)

1. $\frac{3}{5} = 0.6$ $\frac{3}{5} = \frac{6}{10} = 0.6$ 2. $\frac{5}{16}$ 3. $-\frac{3}{20} = -0.15$ $\frac{0.15}{20\overline{)30}}$ → $\frac{1.5}{10} = \frac{15}{100}$

4. $\frac{5}{8}$ $3\overline{)2.0}$ $\frac{0.66}{18\downarrow}$ $\frac{}{20}$ 5. $-\frac{2}{3} = -0.\overline{6}$ $\frac{20\downarrow}{100}$ 6. $-\frac{7}{9}$ $\frac{3}{20} = \frac{15}{100}$

7. In one season, the New England Patriots converted 16 of 20 fourth downs. What part of the time did the Patriots convert on fourth down? (Example 3) $20\overline{)16}$ $\frac{16}{20} \div \frac{4}{4} = \frac{4}{5} = \frac{8}{10} = 0.8$

Replace each ● with <, >, or = to make a true sentence. (Example 4)

8. $0.89 ● \frac{11}{13}$ $\frac{5}{9} \cdot \boxed{\frac{11}{11}} = \frac{55}{99}$ 9. $-\frac{2}{3} < -\frac{3}{5}$ $-0.\overline{6} < -0.6$ 10. $-0.21 ● \frac{1}{5}$ $\frac{3}{4}\boxed{\frac{9}{9}} = \frac{27}{36}$

11. $\frac{5}{9} > \frac{6}{11}$ $\frac{6}{11}\boxed{\frac{9}{9}} = \frac{54}{99}$ 12. $-\frac{9}{15} ● -0.61$ 13. $\frac{3}{4} < \frac{7}{9}$ $\frac{7}{9} \cdot \boxed{\frac{4}{4}} = \frac{28}{36}$

14. Of Nikki's home water usage, $\frac{7}{50}$ comes from lawn watering, and $\frac{3}{20}$ comes from cooking. Does a greater fraction of water usage come from lawn watering or from cooking? (Example 5) $\frac{7}{50} = \frac{140}{1000} < \frac{3}{20} = \frac{150}{1000}$ $\frac{3}{20} = \frac{6}{40}$

15. On his first reading test, Tre answered $\frac{26}{30}$ questions correctly. On his second reading test, he answered $\frac{34}{40}$ questions correctly. On which test did Tre have the better score? (Example 5) $\frac{6+15}{40\ 20}$

$\frac{26}{30} = \frac{13}{15}$ $\frac{34}{40} \cdot \frac{30}{30} = \frac{1020}{1200}$ $\frac{26}{30} \cdot \frac{40}{40} = \frac{1040}{1200}$

$\frac{17}{20}$ BETTER ON FIRST READING TEST

Write each fraction as a decimal. Use a bar to show a repeating decimal. (Examples 1 and 2)

16. $\frac{3}{8}$

17. $\frac{7}{20} = \frac{35}{100} = 0.35$

18. $-\frac{8}{25} = -\frac{32}{100} = -0.32$

19. $-\frac{3}{16}$ -0.1875

20. $\frac{4}{5} = \frac{8}{10} = 0.8$

21. $\frac{9}{25} = \frac{36}{100} = 0.36$

22. $-\frac{1}{8}$

23. $-\frac{7}{16}$

24. $\frac{3}{11}$

25. $\frac{33}{45} = \frac{11}{15} = 0.7\overline{3}$

26. $-\frac{5}{11} = -0.\overline{45}$ $11\overline{)5.0}$ $\frac{44}{60}$ $\frac{55}{50}$ (work: 0.4545)

27. $-\frac{2}{9}$ $-0.\overline{2}$

28. The customer service department resolved 106 of 120 customer complaints in a one-hour time span. To the nearest thousandth, find the resolve rate of the customer service department. (Example 3)

29. In a recent season, Niklas Backstrom of the Minnesota Wild saved 955 out of 1028 shots on goal. To the nearest season thousandth, what part of the time did Backstrom save shots on goal? (Example 3)

$10280 - 1028$

(handwritten) ALMOST 93%
$0.929 = 92.9\%$

$1028\overline{)955.0}$ 0.928
$\frac{9252}{29780}$
$\frac{20564}{92340}$
$\frac{8224}{10160}$

Replace each ● with <, >, or = to make a true sentence. (Example 4)

30. $\frac{6}{15}$ ● 0.4

31. 0.7 ● $\frac{17}{20}$

32. $\frac{5}{6}$ ● $\frac{7}{8}$

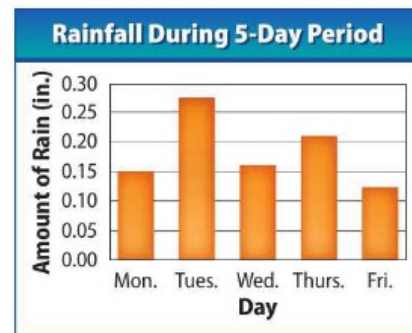33. $\frac{5}{7}$ ● $\frac{10}{14}$

34. $-\frac{2}{9}$ ● $-\frac{1}{4}$

35. $-\frac{1}{8}$ ● $-\frac{1}{10}$

36. $0.\overline{6}$ ● $\frac{5}{9}$

37. $\frac{1}{2}$ ● 0.67

The graph at the right shows the amount of rain, in inches, that fell in a 5-day period. (Example 5)

38. On which days did it rain less than one-fifth of an inch?

39. Did more or less than one-fourth inch of rain fall on Tuesday? Explain.

40. Suppose it rained $\frac{9}{10}$ inch on Saturday. How does this compare to the previous five days?

Rainfall During 5-Day Period

Amount of Rain (in.) — y-axis: 0.00, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30
x-axis (Day): Mon., Tues., Wed., Thurs., Fri.

Replace each ● with <, >, or = to make a true sentence.

41. $-\frac{5}{13}$ ● $-0.\overline{36}$

42. $0.\overline{54}$ ● $\frac{6}{11}$

43. $-5.\overline{42}$ ● $-5\frac{3}{7}$

44. $-\frac{5}{16}$ ● $-\frac{8}{25}$

45. -2.2 ● $-2\frac{2}{7}$

46. $-5\frac{1}{3}$ ● $-5\frac{3}{10}$

47. A carpenter has some bolts that are marked $\frac{1}{2}$, $\frac{5}{16}$, $\frac{3}{32}$, $\frac{3}{4}$, and $\frac{3}{8}$. If all measurements are in inches, how should these bolts be arranged from least to greatest?

Order each group of numbers from least to greatest.

48. -0.29, $-\frac{3}{11}$, $-\frac{2}{7}$

49. $2\frac{3}{5}$, 2.67, $2\frac{2}{3}$

50. $-1.\overline{1}$, $-1\frac{1}{8}$, $-1\frac{1}{10}$

51. $\frac{2}{25}$, $\frac{1}{13}$, 0.089

$$15 \overline{)11.0} \quad \frac{0.7333}{} = 0.7\overline{3}$$

105
50
45
50
45
50

$$16 \overline{)3.0} \quad \frac{0.1875}{}$$

16
140
128
120
112
80

$$15 \overline{)13.0} \quad \frac{0.8}{}$$

10   10280
9   < 9550

$$1028 \overline{)955.0} \quad \frac{0.9289}{}$$

925 2 ↓
29 78 0
2 0 5 6 ↓
92 4 0
8 2 2 4 ↓
1 0 1 6 0

ABOUT
→ 0.929 = 92.9%
ALMOST 93%

$$\frac{24}{30}$$

$$\frac{4}{5}$$

$$
\begin{array}{r}
0.7333 \\
15\overline{)11.0} \\
\underline{10\ 5}\downarrow \\
50 \\
\underline{45}\downarrow \\
50 \\
\underline{45} \\
50 \\
\end{array}
$$

52. The table shows the number of times at bat and hits that players on the Rawson Middle School team had last season. Order the players based on their batting averages from greatest to least.
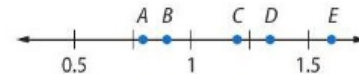(*Hint:* Divide the number of hits by the number of at bats.)

Player	Hits	At Bats
Kristen	35	47
Cho	51	73
Brooke	36	50
Alma	49	65
Jessica	46	60

Write each decimal using bar notation.

53 0.99999…

54. 4.636363…

55. −10.3444…

56. −22.8151515…

57. CCSS **Multiple Representations** Use the number line shown.

    a. Numbers Find a fraction or mixed number that might represent each point on the graph.

    b. Symbols Write an inequality using two of your values.

58. The table shows the number of each type of bead on 3 bracelets that Mrs. Fraser made for a craft show. Which bracelet has the greatest fraction of glass beads? the least?

Mrs. Fraser's Bracelets			
Bead Type	Bracelet 1	Bracelet 2	Bracelet 3
glass	9	10	9
clay	5	5	4
metal	12	18	14

🔥 **H.O.T. Problems** Higher Order Thinking

59. CCSS **Model with Mathematics** Give one example each of real-world situations where it is most appropriate to give a response in fractional form and in decimal form.

60. CCSS **Justify Conclusions** Are there any rational numbers between $0.\overline{4}$ and $\frac{4}{9}$? Explain.

61. CCSS **Identify Structure** A *unit fraction* is a fraction that has 1 as its numerator. Write the four greatest unit fractions that are repeating decimals. Write each fraction as a decimal.

62. CCSS **Use Math Tools** Luke is making lasagna that calls for $\frac{4}{5}$ pound of mozzarella cheese. The store only has packages that contain 0.75- and 0.85-pound of mozzarella cheese. Which of the following techniques might Luke use to determine which package to buy? Justify your selection(s). Then use the technique(s) to solve the problem.

    mental math   *number sense*   *estimation*

63. CCSS **Make a Conjecture** Write the following fractions as decimals: $\frac{1}{9}$, $\frac{23}{99}$, and $\frac{75}{99}$. Make a conjecture about how to express these kinds of fractions as decimals.

64. ⓔ **Building on the Essential Question** Explain how 0.5 and $0.\overline{5}$ are different. Which is greater?

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65. Sherman answered $\frac{4}{5}$ of the multiple-choice questions on his science test correctly. Write this fraction as a decimal.

A 0.4   **C** 0.8

B 0.45   **D** 4.5

66. Which of the following show the fractions $\frac{2}{5}, \frac{3}{8}, \frac{1}{3}, \frac{1}{2},$ and $\frac{5}{12}$ in order from least to greatest?

F $\frac{1}{3}, \frac{3}{8}, \frac{2}{5}, \frac{1}{2}, \frac{5}{12}$ **H** $\frac{1}{3}, \frac{3}{8}, \frac{2}{5}, \frac{5}{12}, \frac{1}{2}$

G $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}, \frac{5}{12}$ **J** $\frac{1}{2}, \frac{5}{12}, \frac{2}{5}, \frac{3}{8}, \frac{1}{3}$

67. The fraction $\frac{7}{9}$ is found between which pair of fractions on a number line?

A $\frac{3}{5}$ and $\frac{3}{4}$ **C** $\frac{7}{10}$ and $\frac{3}{4}$

B $\frac{7}{10}$ and $\frac{4}{5}$ **D** $\frac{3}{5}$ and $\frac{2}{3}$

68. **Short Response** Which item(s) shown in the table have a recycle rate less than one half?

| Material	Fraction Recycled
paper	$\frac{5}{11}$
aluminum cans	$\frac{5}{8}$
glass	$\frac{2}{5}$

Find each product or quotient. 7.NS.2

69. $4(-12)(-5)$ **70.** $-2(42)(3)$ **71.** $-54 \div (-6)$ **72.** $72 \div (-9)$

73. A scuba diver descends from the surface of the lake at a rate of 6 meters per minute. Where will the diver be in relation to the lake's surface after 4 minutes? 6.RP.2

Name the ordered pair for each point graphed at the right. 6.NS.6b

74. E **75.** G

76. C **77.** D

78. A **79.** B
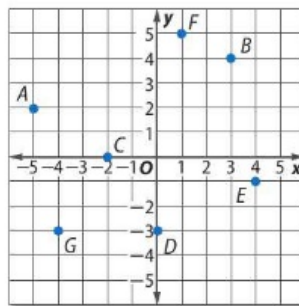
Evaluate each expression if $x = 7$, $y = 3$, and $z = 5$. 6.EE.2

80. $x + y + z$ **81.** $4x - z$

82. $6y - z$ **83.** $9x + 8y$

84. A store sells new and used CDs. Each type of CD sells for the same price. Roberta bought an equal number of new and used CDs. Find the missing values. 6.NS.3

Roberta's Purchases			
Description	Number	Unit Cost	Price
new CD	■	$9.95	$39.80
used CD	■	$5.98	$23.92
		Total:	$63.72

Write each decimal in word form. 5.NBT.3

85. 0.34   **86.** 5.836

87. 0.3   **88.** 1.6