Lesson 3-1

Fractions and Decimals

Interactive Study Guide

See pages 51–52 for:
- Getting Started
- Vocabulary Start-Up
- Notes

Essential Question

What happens when you add, subtract, multiply, and divide rational numbers?

Common Core State Standards

Content Standards
7.NS.2, 7.NS.2d, 8.NS.1, 7.EE.3

Mathematical Practices
1, 3, 4, 5, 7

Vocabulary

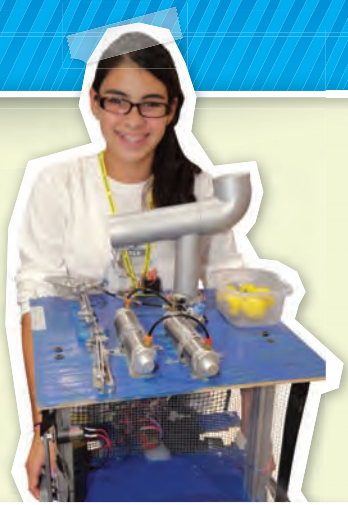
repeating decimal
terminating decimal
bar notation

What You'll Learn

- Write fractions as terminating or repeating decimals.
- Compare fractions and decimals.

Real-World Link

Robotics If you could design your own robot, what would it look like? What would it be able to do? In an annual robot competition, middle school students apply math and science to design, program, and test their own robots. The goal is to make their 'bots outperform the competition!



Jeff Greenberg/Alamy

Write Fractions as Decimals

Some fractions like $\frac{1}{2}$ and $\frac{3}{4}$ can be written as a decimal by making equivalent fractions with denominators of 10, 100, or 1000. However, any fraction $\frac{a}{b}$, where $b \neq 0$, can be written as a decimal by dividing the numerator by the denominator. So, $\frac{a}{b} = a \div b$. The decimal form of a rational number is called a **repeating decimal**.

If the repeating digit is zero, then the decimal is a **terminating decimal**.

Example 1

Tutor

Write $\frac{7}{8}$ as a decimal.

Method 1 Use paper and pencil.

$$
\begin{array}{r}
0.875 \\
8\overline{)7.000} \\
-64 \\
\hline
60 \\
-56 \\
\hline
40 \\
-40 \\
\hline
0
\end{array}
$$

Place the decimal point. Annex zeros and divide as with whole numbers.

Division ends when the repeating digit is 0.

Method 2 Use a calculator.

7 ÷ 8 ENTER **0.875**

Using either method, $\frac{7}{8} = 0.875$.

Got It? Do these problems to find out.

Write each fraction as a decimal.

1a. $\frac{4}{5}$ **0.8**

1b. $\frac{3}{16}$ **0.1875**

Vocabulary Link
Terminating
Everyday Use bringing to an end
Math Use a decimal whose digits end

Not all fractions have repeating digits that are zero. Sometimes a nonzero digit or a group of digits repeats without end in the quotient

$\frac{1}{6} \rightarrow 6\overline{)1.000}$ → 0.166

-6
40
-36
40
-36
4

The digit 6 repeats.

The remainder after each step is 4.

Check 1 ÷ 6 ENTER **0.1666666667** ✔ The last digit is rounded.

You can indicate that the digit 6 repeats by annexing dots. So,

$\frac{1}{6} = 0.1666666666\ldots$. This decimal is called a repeating decimal.

Repeating decimals have a pattern in their digits that repeats without end. **Bar notation** is a bar or line placed over the digit(s) that repeats. The table shows some examples of repeating decimals and their bar notations.

Decimal	Bar Notation
0.166666…	$0.1\overline{6}$
0.353535…	$0.\overline{35}$
12.6888888…	$12.6\overline{8}$
5.714285714285…	$5.\overline{714285}$

Example 2

Tutor

Write each fraction as a decimal. Use a bar to show a repeating decimal.

a. $\frac{5}{12}$

$\frac{5}{12} \rightarrow 12\overline{)5.0000...}$ → 0.4166... The digit 6 repeats.

So, $\frac{5}{12} = 0.41\overline{6}$.

b. $-\frac{2}{11}$

$-\frac{2}{11} \rightarrow 11\overline{)2.0000...}$ → 0.1818... The digits 18 repeat.

So, $-\frac{2}{11} = -0.\overline{18}$.

Got It? **Do these problems to find out.**

2a. $-\frac{5}{6}$ **$-0.83\overline{}$**

2b. $\frac{7}{9}$ **$0.\overline{7}$**

It is helpful to memorize these fraction-decimal equivalents.

Concept Summary **Fraction-Decimal Equivalents**

$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.\overline{3}$	$\frac{1}{4} = 0.25$	$\frac{1}{5} = 0.2$	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$
$\frac{2}{3} = 0.\overline{6}$	$\frac{3}{4} = 0.75$	$\frac{2}{5} = 0.4$	$\frac{3}{5} = 0.6$	$\frac{4}{5} = 0.8$	$\frac{5}{6} = 0.8\overline{3}$

Example 3

According to the USDA, teenage boys should consume an average of 2700 Calories per day. About 360 Calories should come from milk. To the nearest hundredth, what part of a teenage boy's total Calories should come from milk?

Divide the number of Calories that should come from milk, 360, by the number of total Calories, 2700.

360 ÷ 2700 ENTER **0.133**... or $0.1\overline{3}$

Look at the digit to the right of the thousandths place. Round down since $3 < 5$.

Milk should be 0.13 of the daily Calories consumed by a teenage boy.

Got It? Do this problem to find out.

3. In a recent Masters Tournament, Zach Johnson's first shot landed on the fairway 45 out of 56 times. To the nearest thousandth, what part of the time did his shot land on the fairway? **0.804**

Compare Fractions and Decimals

It may be easier to compare numbers when they are written as decimals.

Example 4

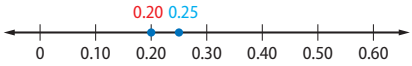
Replace each ● with <, >, or = to make a true sentence.

a. $\frac{1}{4}$ ● **0.2**

$\frac{1}{4}$ ● 0.2 Write the sentence.

0.25 ● 0.2**0** Write $\frac{1}{4}$ as a decimal. Annex a zero to 0.2.

0.25 > 0.20 In the hundredths place, 5 > 0.

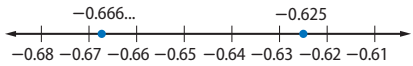


Check Since 0.20 is to the left of 0.25 on the number line, $\frac{1}{4} > 0.2$.

b. $-\frac{5}{8}$ ● $-\frac{6}{9}$

Write the fractions as decimals and then compare the decimals.

$-\frac{5}{8} = -0.625$ $-\frac{6}{9} = -0.666\ldots$ or $-0.\overline{6}$



Since -0.625 is to the right of $-0.\overline{6}$ on the number line, $-\frac{5}{8} > -\frac{6}{9}$.

Got It? Do these problems to find out.

4a. $\frac{7}{8}$ ● 0.87 **>**

4b. $-\frac{7}{15}$ ● $-\frac{5}{12}$ **<**

Comparing Decimals

When comparing two decimals, compare the digits in the same place-value position.

Example 5

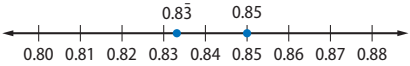
Use a Graph
You can use a graph to visualize data, analyze trends, and make predictions. In this example, you can compare the decimals on a number line.

Thirty out of 36 seventh graders and 34 out of 40 eighth graders participated in a marathon for charity. Which class had a greater fraction participating in the marathon?

Write each fraction as a decimal.
Then compare the decimals.

seventh graders: $\frac{30}{36} = 0.8\overline{3}$

eighth graders: $\frac{34}{40} = 0.85$



On a number line, $0.8\overline{3}$ is to the left of 0.85. Since $0.8\overline{3} < 0.85$, $\frac{30}{36} < \frac{34}{40}$.

So, a greater fraction of eighth graders participated in the marathon.

Got It? **Do this problem to find out.**

5. Over the weekend, $\frac{16}{28}$ of the eighth grade girls and $\frac{19}{30}$ of the eighth grade boys went to see a new comedy movie. Did a greater fraction of girls or boys see the movie? **boys**

Guided Practice

Write each fraction as a decimal. Use a bar to show a repeating decimal. (Examples 1 and 2)

1. $\frac{3}{5}$ **0.6**

2. $\frac{5}{16}$ **0.3125**

3. $-\frac{3}{20}$ **-0.15**

4. $\frac{5}{8}$ **0.625**

5. $-\frac{2}{3}$ **$-0.\overline{6}$**

6. $-\frac{7}{9}$ **$-0.\overline{7}$**

7. In one season, the New England Patriots converted 16 of 20 fourth downs. What part of the time did the Patriots convert on fourth down? (Example 3) **0.8**

Replace each ● with <, >, or = to make a true sentence. (Example 4)

8. $0.89 \bullet \frac{11}{13}$ **>**

9. $-\frac{2}{3} \bullet -\frac{3}{5}$ **<**

10. $-0.21 \bullet \frac{1}{5}$ **<**

11. $\frac{5}{9} \bullet \frac{6}{11}$ **>**

12. $-\frac{9}{15} \bullet -0.61$ **>**

13. $\frac{3}{4} \bullet \frac{7}{9}$ **<**

14. Of Nikki's home water usage, $\frac{7}{50}$ comes from lawn watering, and $\frac{3}{20}$ comes from cooking. Does a greater fraction of water usage come from lawn watering or from cooking? (Example 5) **cooking**

15. On his first reading test, Tre answered $\frac{26}{30}$ questions correctly. On his second reading test, he answered $\frac{34}{40}$ questions correctly. On which test did Tre have the better score? (Example 5) **test 1**