Lesson 4-4

Scientific Notation

Essential Question

Why is it useful to write numbers in different ways?

CCSS Common Core State Standards

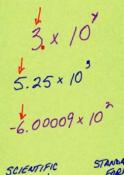**Content Standards**
8.EE.1, 8.EE.3, 8.EE.4

Mathematical Practices
1, 3, 4, 7

Vocabulary

standard form
scientific notation

What You'll Learn

- Express numbers in standard form and in scientific notation.
- Compare and order numbers written in scientific notation.

Real-World Link

Space Earth is the third planet from the Sun in our solar system. Because Earth's rotation about the Sun is not circular, the maximum distance between Earth and the Sun is about 95 million miles and the minimum distance is about 91 million miles.

Key Concept	Scientific Notation
Words	A number is expressed in scientific notation when it is written as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.
Symbols	$a \times 10^n$, where $1 \le a < 10$ and n is an integer.
Examples	$3{,}500{,}000 = 3.5 \times 10^6$ $\qquad$ $0.00004 = 4 \times 10^{-5}$

Numbers that do not contain exponents are written in **standard form**. However, when you deal with very large numbers like 12,760,000 or very small numbers like 0.00001276, it can be difficult to keep track of the place value. A number that is expressed as a product of a factor and a power of 10 is written in **scientific notation**.

When a number is expressed in scientific notation the exponent tells you how many places to move the decimal point.

Example 1

Express each number in standard form.

a. 2×10^3
$2 \times 10^3 = 2000$ Move the decimal point 3 places to the right.

b. 6.8×10^5
$6.8 \times 10^5 = 680{,}000$ Move the decimal point 5 places to the right.

c. 3.25×10^{-4}
$3.25 \times 10^{-4} = 0.000325$ Move the decimal point 4 places to the left.

Got It? Do these problems to find out.

1a. 4×10^2 **1b.** 5.94×10^7 **1c.** 1.3×10^{-3}

(handwritten notes in margin:)

3×10^4

5.25×10^5

-6.00009×10^2

SCIENTIFIC NOTATION   STANDARD FORM

$5.25 \times 10^3 = 5250$

$6.313 \times 10^{-3} = 0.006313$

$0.006.313$

$BASE_{10}$ HUNDREDS   TENS   ONES   •   TENTHS   HUNDREDTHS   THOUSANDTHS

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \bullet\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$

$BASE_{3}$ NINES   THREES   ONES   •   THIRDS   NINETHS   TWENTYSEVENTHS

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \bullet\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$

$BASE_{10} \rightarrow 11$ $BASE_{3} \rightarrow 102$

16   121

When expressing a number in scientific notation, the sign of the exponent can be determined by evaluating the number in standard form. If a number in standard form is greater than or equal to 1, then the exponent is *positive*. If a number is between 0 and 1, then the exponent is *negative*.

Example 2

Tutor

Express each number in scientific notation.

a. 4,000,000

$4{,}000{,}000 = 4 \times 10^6$

The decimal point moves 6 places.
The exponent is positive.

b. 0.072

$0.072 = 7.2 \times 10^{-2}$

The decimal point moves 2 places.
The exponent is negative.

Got It? Do these problems to find out.

2a. 900   **2b.** 18,900   **2c.** 0.000064

$900 = 9 \times 10^2$

$18{,}900 = 1.89 \times 10^4$

One way to estimate a very large or a very small number is to express it in the form of a single digit times an integer power of 10. For example, the population of the United States in 2010 was 308,745,538. The number 3×10^8 is an estimate of that number.

Example 3

Tutor

The population of Kansas is 2,853,118 people. Write an estimation in scientific notation for the population.

$2{,}853{,}118 \approx 3{,}000{,}000$ Estimate.
$3{,}000{,}000 = 3 \times 10^6$ Write in scientific notation.

The population of Kansas is about 3×10^6 people.

Got It? Do these problems to find out.

Estimate each value using scientific notation.

3a. 3,612,500 cm   **3b.** 0.000000251 ft   **3c.** 4.215×10^{-3} kg

Example 4

Tutor

STEM **The space shuttle traveled at about 8 kilometers per second. At this rate, the shuttle would take about 4.5×10^4 seconds to fly to the moon. Is it more appropriate for a newspaper to report this time as about 4.5×10^4 seconds or about 12.5 hours? Explain your reasoning.**

The measure 12.5 hours is more appropriate. The number 4.5×10^4 seconds is very large, so choosing the larger unit of measure is more meaningful.

Got It? Do this problem to find out.

4. A dime is about 5.875×10^{-3} foot in diameter. Is it more appropriate to report that the diameter of a dime is 5.875×10^{-3} foot or 7.05×10^{-1} inch? Explain your reasoning.

Compare and Order Numbers

To compare and order numbers in scientific notation, first compare the exponents. With positive numbers, the number with a greater exponent is greater. If the exponents are the same, compare the factors.
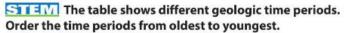
Example 5

Power of 10

When writing a number in scientific notation, the power of 10 is determined by the direction and number of places you move the decimal point.

STEM The table shows different geologic time periods. Order the time periods from oldest to youngest.

Geologic Time Periods	
Period	Number of Years Ago
Jurassic	2.08×10^8
Silurian	4.38×10^8
Tertiary	6.64×10^7
Triassic	2.45×10^8

Step 1 Order the numbers according to their exponents.

The Tertiary period has an exponent of 7. So, it is the youngest period.

Step 2 Order the numbers with the same exponent by comparing the factors.

$$4.38 \quad > \quad 2.45 \quad > \quad 2.08$$

Silurian	Triassic	Jurassic
↓	↓	↓

So, $4.38 \times 10^8 > 2.45 \times 10^8 > 2.08 \times 10^8$.

The time periods ordered from oldest to youngest are Silurian, Triassic, Jurassic, and Tertiary.

Got It? Do this problem to find out.

5. **STEM** Approximately 1.372×10^7 square kilometers of Antarctica and about 1.834×10^6 square kilometers of Greenland are covered by an ice cap. Which land mass has a greater area covered by ice?

Guided Practice

Express each number in standard form. (Example 1)

1. 4.16×10^3
2. 3.2×10^{-2}
3. 1.075×10^5

Express each number in scientific notation. (Example 2)

4. 1,600,000
5. 135,000
6. 0.008

Estimate each value using scientific notation. (Example 3)

7. 0.000007109 kg
8. 3.7085×10^{14} mL
9. 18,900,435 cm

10. If you could walk at the rate of about 1 mile every 20 minutes without stopping, it would take about 1.4×10^2 hours to walk from Columbus, Ohio, to Washington, D.C. Is it more appropriate to report the time as 1.4×10^2 hours or 8.4×10^3 minutes? Explain. (Example 4)

11. Order 3.4×10^2, 3.5×10^2, 3.7×10^{-2}, and 400 from least to greatest. (Example 5)

Express each number in standard form. (Example 1)

12. 6.89×10^4

13. 1.5×10^{-4}

14. 2.3×10^{-5}

15. 9.51×10^{-3}

16. 3.062×10^6

17. 7.924×10^2

18. A dollar bill is approximately 1.09×10^{-2} centimeter thick. Write 1.09×10^{-2} in standard form.

19. It is estimated that more than 1.71×10^{11} E-mails are sent each day around the world. Write 1.71×10^{11} in standard form.

Express each number in scientific notation. (Example 2)

20. 700,000

21 32,000,000

22. 0.045

23. 0.000918

24. 1,000,000

25. 0.006752

Estimate each value using scientific notation. (Example 3)

26. 0.00000095 centimeter

27. 8.375×10^{-23} pound

28. 56,300,001 miles

29. The distance between Earth and the Moon is about 3.84×10^5 kilometers. Estimate this distance using scientific notation. (Example 3)

30. The usual growth rate of human hair is 3.3×10^{-4} meter per day. Is it more appropriate to report the rate as 3.3×10^{-4} meter per day or 0.33 millimeter per day? Explain your reasoning. (Example 4)

31. One ounce of a certain cheese has 219 milligrams of calcium. Is it more appropriate to include on the nutrition label that the cheese has 2.19×10^{-4} kilogram of calcium or 219 milligrams of calcium? (Example 4)

Order each set of numbers from least to greatest. (Example 5)

32. 2.4×10^2, 2.45×10^{-2}, 2.45×10^2, 2.4×10^{-2}

33. 2.81×10^4, 2805, 2.08×10^5, 3.2×10^4, 3.024×10^2

34. 5.9×10^6, 5.9×10^4, 5.01×10^5, 5.1×10^{-3}

35. 9,562,301, 9.05×10^{-6}, 9.5×10^6, 905,000

36. List the states in the table at the right from least to greatest production of maple syrup. (Example 5)

37. STEM A sheet of gold leaf is approximately 1.25×10^{-5} centimeter thick.

  a. Write the value of the thickness as a decimal.   *0.0000125 cm*

  b. Use the formula $V = \ell wh$ to find the volume in cubic meters of a sheet of gold that is 2 meters wide and 5 meters long.

$\dfrac{0.0000125}{100} = 0.000000125$

$0.000000125 \times 2 \times 5 = 0.000000125 \times 10$

$0.00000125\, cm^3$

State	Amount of Syrup Produced (L)
Maine	1.10×10^6
New Hampshire	3.14×10^5
New York	9.65×10^5
Vermont	1.89×10^6
Wisconsin	3.79×10^5

$1m = 100\ cm$ $m \times 100 = cm$ $2m = 200\ cm$

$1\ cm = 0.01\ m$ $\dfrac{cm}{100} = m$ $5m = 500\ cm$

Replace each ● with <, >, or = to make a true statement.

38. 5.72×10^8 ● 5.8×10^8

39. $35,400$ ● 35.4×10^3

40. 0.042 ● 4.2×10^{-3}

41. 5×10^5 ● $5,000,000$

42. $27,000$ ● 2.76×10^3

43. 6.4×10^{-5} ● 0.000649

44. **CCSS** **Justify Conclusions** A news article reported the population of Illinois to be 12,869,257 and the estimated population of Indiana to be 6×10^6.

   a. Estimate the number of people living in Illinois. Express the number in scientific notation.

   b. Which state, Illinois or Indiana, has the greater population? Explain your reasoning.

   c. Estimate the combined population of the two states. Express the number in scientific notation.

45. **STEM** The Moon travels around the Earth at a speed of about 3.68×10^3 kilometers per hour. If the Moon orbits the Earth every 27.3 days, about how far does it travel in one orbit around the Earth?

   3680

   ↳ 655.2 HOURS

   3680(655.2)= 2,411,136 KM

46. The speed of light is about 3×10^5 kilometers per second. The distance between Earth and the Moon is about 3.84×10^5 kilometers. Find how long it would take for light to travel from Earth to the Moon.

47. In a recent year, U.S. route 59 in the Houston area averaged approximately 338,510 vehicles per day. About how many vehicles was this during the entire year? Write the number in scientific notation. Verify your solution by using estimation.

   338,510(365) = 123,556,150

   340,000(350) = 119,000,000 → 1.19×10^8

H.O.T. Problems *Higher Order Thinking*

48. **CCSS** **Identify Structure** Write two numbers in scientific notation with different exponents. Then find the sum, difference, product, and quotient of the two numbers. Write the answers in scientific notation.

49. **CCSS** **Persevere with Problems** A *googol* is a number that is 1 followed by 100 zeros. A *centillion* is a number that is 1 followed by 303 zeros. Write each of these numbers in scientific notation.

50. **CCSS** **Justify Conclusions** Miami is the second most populous city in Florida.

   a. Which number better describes the population of Miami: 3.8×10^4 or 3.8×10^6? Explain.

   b. Express Miami's population in another form.

   c. Which notation is best to use when describing population? Explain.

   $3 \times 10^2 = 300$

51. **CCSS** **Persevere with Problems** Which number is twice as great as 3×10^2: 6×10^2, 3×10^4, or 6×10^4? Explain.

   $6 \times 10^2 = 600$
   $3 \times 10^4 = 30,000$
   $6 \times 10^4 = 60,000$

52. **Building on the Essential Question** Your friend thinks 7.8×10^3 is greater than 6.5×10^2 because $7.8 > 6.5$. Explain why your friend's reasoning is incorrect.

53. The slowest land mammal is the three-toed sloth, which moves 0.07 mile per hour. Which expression represents this number?

   A 7×10^{-3} **C** 7×10^{2}

   B 7×10^{-2} **D** 7×10^{3}

54. The distance from Earth to the sun is about 9.5×10^{7} miles. Which of the following represents this distance?

   F 9,500,000 mi

   G 95,000,000 mi

   H 950,000,000 mi

   J 9,500,000,000 mi

55. **Short Response** The weight of a fruit fly is about 1.3×10^{-4} pound. Estimate the weight of a fruit fly. Express the weight in the form of a single digit times an integer power of 10.

56. **Short Response** A 45-acre farm produces 366,400 pounds of avocados per year. Write an expression, in scientific notation, for the number of pounds of avocados produced per year.

CCSS   **Common Core Review**

57. **STEM** Which type of molecule in the table has a greater mass? How many times greater is it than the other type? **8.EE.1**

Molecule	Mass (kg)
penicillin	10^{-18}
insulin	10^{-23}

Find each product or quotient. Express using exponents. **8.EE.1**

58. $a \cdot a^5$

59. $(n^4)(n^4)$

60. $-3x^2(4x^3)$

61. $\dfrac{3^8}{3^{-5}}$

62. Use the table to write an expression that shows how many bushels of green beans were picked. Simplify the expression. **7.EE.1**

63. Translate *eight times the product of six times a number, all divided by 2* into an algebraic expression. Then simplify the expression. **6.EE.2a**

Bushels of Green Beans Picked	
Acre A	40
Acre B	38
Acre C	g

64. **Financial Literacy** Of Quincy's monthly paycheck, $\frac{7}{25}$ goes to pay his mortgage, and $\frac{9}{20}$ goes to pay his college loans. Does a greater fraction of his paycheck go to paying his mortgage or his college loans? **8.NS.1**

Write each expression using a positive exponent. **8.EE.1**

65. 3^{-2}

66. h^{-8}

67. $(-5)^{-1}$

68. x^{-12}

69. 5^{-3}

70. $(-7)^{-4}$

Write each expression using exponents. **8.EE.1**

71. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

72. $(6 \cdot 6 \cdot 6) \cdot 6$

73. $3 \cdot 2 \cdot 3 \cdot 2 \cdot 2$

74. $7 \cdot 7 \cdot 8 \cdot 8 \cdot 8 \cdot 7$

75. $10 \cdot 5 \cdot 10 \cdot 5 \cdot 5 \cdot 5$

76. $(2 \cdot 2) \cdot 2 \cdot 2 \cdot 2$

ISG **Interactive Study Guide**

See page 81 for:
• Mid-Chapter Check

SCIENTIFIC NOTATION   STANDARD FORM

$$9. \times 10^1 = 90$$

$$5. \times 10^2 = 500$$

$$5. \times 10^{-2} = 0.05$$

$$0.05$$

$$19 \times 10^2 = 1900$$
$$1.9 \times 10^3$$

$$5.6 \times 10^5$$

X