Lesson 5-6

Graphing Proportional Relationships

Interactive Study Guide

See pages 109–110 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

How can you identify and represent proportional relationships?

Common Core State Standards

Content Standards
7.RP.2, 7.RP.2a, 7.RP.2b, 7.RP.2d, 8.EE.5

Mathematical Practices
1, 3, 4

What You'll Learn

- Identify proportional relationships.
- Analyze proportional relationships.

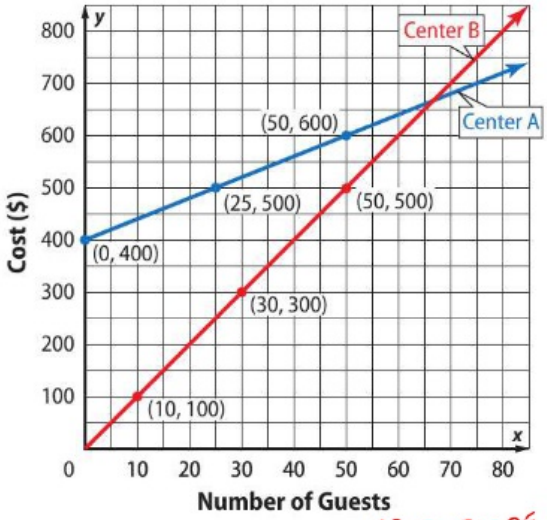
Real-World Link

Parties Some birthday traditions, like having parties and giving gifts, are similar throughout the world. But many cultures celebrate significant birthdays in other ways, such as by flying flags or having a dance. The age at which a child moves into adulthood, or *coming of age*, varies by culture. The celebrated age can be 12, 13, 15, 16, or even 18.

Identify Proportional Relationships

Another way to determine whether two quantities are proportional is to graph the quantities on the coordinate plane. If the graph of the two quantities is a straight line through the origin, then the two quantities are proportional.

The cost of renting Center A and Center B for a party is shown in the graph below. $\frac{20}{1} \neq \frac{12}{1}$



$\frac{500}{25} = \frac{20}{1}$

$\frac{600}{50} = \frac{12}{1}$

$\frac{300}{30} = 10$

$\frac{500}{50} = 10$

CONSTANT OF PROPORTIONALITY (K)

$K = \frac{y}{x}$

$y = Kx$

10 → SLOPE, CONSTANT OF PROPORTIONALITY

For Center A, the rate is not constant, so the relationship between the cost and the number of guests is nonproportional. Notice that the graph for Center A is a *straight line* that *does not* pass through the origin.

For Center B, the rate is constant, so the relationship between the cost and the number of guests for Center B is proportional. Notice that the graph for Center B is a *straight line* that *does* pass through the origin.

$\frac{10}{2} = 5$ $\quad 5 \cdot 2 = 10$ How many twos go into ten?

Five twos go into ten.

$\frac{400}{0} =$ $\quad 0 \cdot$ [NOTHING WORKS] $= 400$ How many zeros go into 400?

$\frac{400}{0}$ = UNDEFINED

Example 1

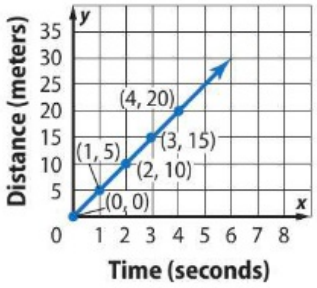
Tutor

Determine whether each relationship is proportional by graphing on the coordinate plane. Explain your reasoning.

a. The black mamba is the fastest snake in the world. The table shows the distance the snake travels for several different times. Is the distance the snake travels proportional to the time?

Watch Out!
Since $\frac{0}{0}$ is undefined, it is not included in the list of rates.

Time (s)	0	1	2	3	4
Distance (m)	0	5	10	15	20



Graph the ordered pairs on the coordinate plane. Then connect the ordered pairs.

The line passes through the origin and is a straight line. So, the distance traveled in meters is proportional to the time in seconds.

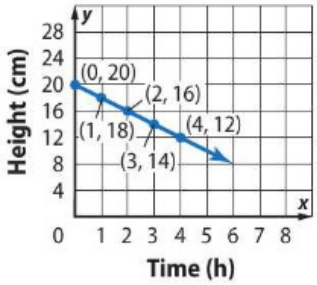
Check The ratios are constant. $\frac{5}{1}, \frac{10}{2} = \frac{5}{1}, \frac{15}{3} = \frac{5}{1}, \frac{20}{4} = \frac{5}{1}$

The relationship is proportional. ✓

b. A candle is 20 centimeters tall. It burns at a rate of 2 centimeters per hour. Is the height of the candle proportional to the number of hours it burns?

Make a table to find the height of the candle after 0, 1, 2, 3, and 4 minutes.

Time (h)	0	1	2	3	4
Height (cm)	20	18	16	14	12



Graph the ordered pairs on the coordinate plane. Then connect the ordered pairs.

$\frac{8}{4} = \frac{2}{1}$

The graph is a straight line but does not pass through the origin. So, the height of the candle is nonproportional to the numberof hours it burns.

$\frac{12}{4} = \frac{3}{1}$

$\frac{6}{2} = \frac{3}{1}$

Check The ratios are not constant. $\frac{18}{1}, \frac{16}{2} = \frac{8}{1}, \frac{14}{3} = 4\frac{2}{3}, \frac{12}{4} = \frac{3}{1}$

$\frac{6}{2} = \frac{3}{1}$

$\frac{18}{6} = \frac{3}{1}$

The relationship is nonproportional. ✓

Got It? Do these problems to find out.

$\frac{24}{8} = \frac{3}{1}$

Determine whether the cost is proportional to the number of items in each relationship by graphing on the coordinate plane. Explain. See Answer Appendix.

$\frac{30}{10} = \frac{3}{1}$

1a. is not proportional

1b. is proportional

1a.

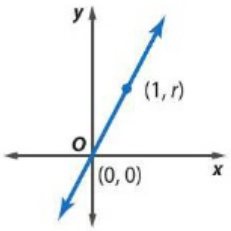
Number of Tickets	2	4	6	8	10
Cost ($)	6	8	10	12	14

1b.

Number of Hotdogs	2	4	6	8	10
Cost ($)	6	12	18	24	30

Analyze Proportional Relationships

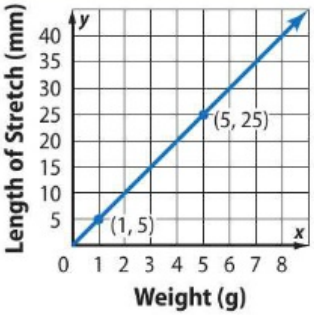
When two quantities are proportional, you can use a graph of the quantities to find the constant of proportionality and to analyze points on the graph. The graph of every proportional relationship passes through the point (0, 0). The point (1, *r*) tells you the constant of proportionality, or the unit rate *r*.



Example 2

Tutor

The length of the stretch (in millimeters) of a spring is proportional to the weight (in grams) attached to the end of the spring as shown in the graph.



a. Find and interpret the constant of proportionality.

Use the point (5, 25) on the graph.

$$\frac{\text{length of stretch (mm)}}{\text{weight (g)}} = \frac{25}{5}$$

$$= \frac{5}{1} \text{ or } 5$$

The constant of proportionality, or unit rate, is 5 millimeters of stretch per gram of weight attached.

b. Explain what the points (0, 0), (1, 5), and (5, 25) represent.

The point (0, 0) represents the length of the stretch of the spring, 0 millimeters, when no weight is attached.

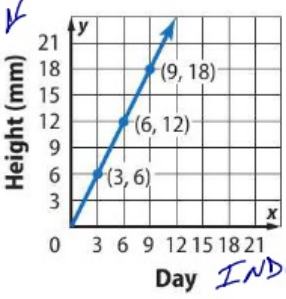
The point (1, 5) represents the length of the stretch of the spring, 5 millimeters, when a one-gram weight is attached.

The point (5, 25) represents the length of the stretch of the spring, 25 millimeters, when a five-gram weight is attached.

DEPENDENT VARIABLE

Got It? Do this problem to find out.

2. Keith plants a seed. Every three days after the seed sprouts he measures the height of the plant. The graph shows his results.



Y / X

INDEPENDENT VALUE

a. Find and interpret the constant of proportionality.

2a. 2; the plant grew 2 mm per day

b. Explain what the points (0, 0), (1, 2), and (6, 12) represent. **After 0 days, the plant grew 0 mm; after 1 day the plant grew 2 mm; after 6 days, the plant grew 12 mm.**

Guided Practice

Determine whether each relationship is proportional by graphing on the coordinate plane. Explain your reasoning. (Example 1) **1–3. See Answer Appendix.**

1.

Time (min)	4	6	8	10
Distance (ft)	5	10	15	20

2.

Number of Gallons	1	2	3	4
Number of Quarts	4	8	12	16

3. The formula for the area A of all parallelograms with a base of 2 centimeters is $A = 2h$, where h is the height in centimeters. Determine whether the area of all parallelograms with a base of 2 centimeters is proportional to the height in centimeters. Explain your reasoning. (Example1)

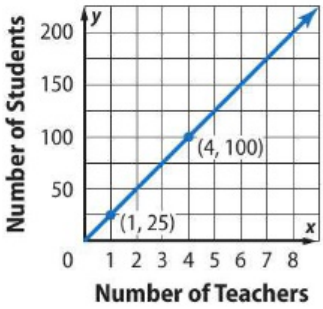
$\frac{6}{3} = \frac{2}{1}$ $\frac{10}{5} = \frac{2}{1}$ $\frac{20}{10} = \frac{2}{1}$

h	A
3	2(3) = 6
5	2(5) = 10
10	2(10) = 20

4. The number of students on a school trip is proportional to the number of teachers as shown in the graph. (Example 2)

a. Find and interpret the constant of proportionality. **25; There are 25 students per teacher.**

b. Explain what the points (0, 0), (1, 25) and (4, 100) represent. **0 teachers would have 0 students on the trip, 1 teacher would have 25 students on the trip, and 4 teachers would have 100 students on the trip.**



Independent Practice

Go online for Step-by-Step Solutions

Determine whether each relationship is proportional by graphing on the coordinate plane. Explain your reasoning. (Example 1) **5–7. See Answer Appendix.**

5

Soap (mL)	3	6	9	12
Water (L)	2	4	6	8

6.

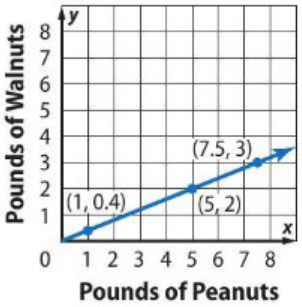
Number of Pizzas	2	4	6	8
Total Cost ($)	22	42	62	82

7. The cost of 3-D movie tickets is $12.50 per ticket. Determine whether the cost is proportional to the number of tickets by graphing the relationship on the coordinate plane. Explain your reasoning. (Example 1)

8. The number of pounds of walnuts in a nut mix is proportional to the number of pounds of peanuts as shown in the graph. (Example 2)

a. Find and interpret the constant of proportionality.

b. Explain what the points (0, 0), (1, 0.4) and (7.5, 3) represent.



8a. 0.4; For every pound of peanuts, there is 0.4 pound of walnuts.

8b. For 0 pound of peanuts there are 0 pounds of walnuts, for 1 pound of peanuts there is 0.4 pound of walnuts, and for 7.5 pounds of peanuts there are 3 pounds of walnuts.