Please solve in your spiral

Greg Biffle won the Samsung/Radio Shack 500 NASCAR race at the Texas Motor Speedway in April 2005. His average speed was 130 miles per hour. How many feet per second is this? Round to the nearest hundredth.

$$\frac{130\text{ mi}}{1\text{ HR}} \cdot \frac{686{,}400\text{ ft}}{130\text{ mi}} = \frac{686{,}400\text{ ft}}{1\text{ HR}} \cdot \frac{1\text{ HR}}{3{,}600\text{ SEC}} = \frac{686{,}400\text{ ft} \div 100}{3{,}600\text{ SEC} \div 100}$$

$130(5280)$

$$36\overline{)686{,}4} \quad 190.\overline{6}$$

190.67 ft/sec

Lesson 5-5
Proportional and Nonproportional Relationships

Interactive Study Guide

See pages 105–106 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

How can you identify and represent proportional relationships?

Common Core State Standards

Content Standards
7.RP.2, 7.RP.2a, 7.RP.2b, 7.RP.2c

Mathematical Practices
1, 3, 4

Vocabulary

proportional
constant of proportionality
nonproportional

What You'll Learn

- Identify proportional and nonproportional relationships in tables and graphs.
- Describe a proportional relationship using an equation.

Real-World Link

Dances A middle school student advisory council is planning a spring dance with a Glow-in-the-Dark theme. The "tickets" will be glow-in-the-dark wristbands. Students on the council found the cost of the wristbands from two companies. Proportions can be used to compare the costs.

Identify Proportional Relationships

Two quantities are **proportional** if they have a constant ratio or rate. The constant ratio is called the **constant of proportionality**.

$\frac{\text{Total Cost (\$)}}{\text{Number of Wristbands}} = \frac{6}{4} = \frac{9}{6} = \frac{12}{8} = \frac{3}{2}$ $\frac{\text{Total Cost (\$)}}{\text{Number of Necklaces}} = \frac{6}{4} \neq \frac{10}{5} \neq \frac{15}{6}$

$\frac{6}{4} : \frac{3}{2}$

$\frac{10}{5} : \frac{2}{1}$

The total cost is proportional to the number of wristbands purchased. The constant of proportionality is $\frac{3}{2}$. However, the rates for the necklaces are not constant. For relationships in which the ratios or rates are *not* constant, the two quantities are said to be **nonproportional**.

Example 1

Tutor

Determine whether the cost of coffee is proportional to the number of pounds. If the relationship is proportional, identify the constant of proportionality. Explain your reasoning.

Coffee (pounds)	Cost (dollars)
1	3
2	6
3	9
4	12

Write the rate of cost to pounds of coffee for each column in the table. Simplify each fraction.

$\frac{3}{1}$ $\frac{6}{2} = \frac{3}{1}$ $\frac{9}{3} = \frac{3}{1}$ $\frac{12}{4} = \frac{3}{1}$ All the rates are equal.

The rates are equal, so the cost is proportional to the number of pounds of coffee. The constant of proportionality is 3.

Got It? Do this problem to find out.

1. Determine whether the number of legs are proportional to the number of spiders. If the relationship is proportional, identify the constant of proportionality. Explain your reasoning.

Number of Spiders	1	2	3	4
Number of Legs	8	16	24	32

PROPORTIONAL

$\frac{1}{2} = \frac{2}{4} = \frac{50}{100}$

NONPROPORTIONAL

$\frac{2}{5} \neq \frac{4}{11}$

MILK

$3/GAL.

GAL	$
5	15
3	9
1	3
0	0

$\frac{\$}{GAL}$

$\frac{15}{5} = 3$

$\frac{9}{3} = 3$

$\frac{3}{1} = 3$

PROPORTIONAL RELATIONSHIP

MILK CLUB

CLUB DUES $5/MO. ← FIXED COST

MILK IS $6/GAL.

GAL	$	
5	35	$\frac{35}{5} = 7$
3	23	$\frac{23}{3} = > 7$
1	11	
0	5	$\frac{11}{1} = 11$

NON PROPORTIONAL RELATIONSHIP

Example 2

Tutor

Determine whether the distance is proportional to the time traveled. If the relationship is proportional, identify the constant of proportionality. Explain your reasoning.

Time (hours)	Distance (miles)
1	50
2	70
3	~~90~~ 150

Write the rate of distance to time for each hour in simplest form.

$\frac{50}{1}$ $\frac{70}{2} = \frac{35}{1}$ $\frac{90}{3} = \frac{30}{1}$ The rates are not equal.

The distance is *not* proportional to the time.

Got It? Do this problem to find out.

2. Determine whether the number of ice cubes is proportional to the number of drinks. If the relationship is proportional, identify the constant of proportionality. Explain your reasoning.

Drinks	1	2	3
Ice Cubes	6	14	22

$\frac{1}{6} \neq \frac{1}{7}$

$\frac{1}{6}$ $\frac{2}{14} = \frac{1}{7}$

Use Proportional Relationships

Constant of Proportionality
The constant of proportionality is also called the unit rate.

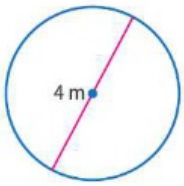
Proportional relationships can also be described using equations of the form $y = kx$, where k is the constant ratio or the constant of proportionality.

Real World

Example 3

Tutor

A circle's circumference is proportional to its diameter. Use the figure to find the constant of proportionality. Then write an equation relating the circumference of the circle to its diameter. What is the circumference of a circle with a 6-inch diameter?



$C = 12.56$ m

Find the constant of proportionality.

$\frac{\text{circumference}}{\text{diameter}} = \frac{12.56}{4}$ or 3.14

Words	The circumference is about 3.14 times the diameter.
Variable	Let C = circumference and d = diameter.
Equation	$C = 3.14d$

$C = 3.14d$ Write the equation.
$= 3.14(6)$ Replace d with 6.
$= 18.84$ Multiply.

The circumference is about 18.84 inches.

Got It? Do this problem to find out.

3. The cost for $2\frac{1}{2}$ pounds of meat is \$7.20. Find the constant of proportionality. Then write an equation relating cost to pounds. How much will 4 pounds cost?

Guided Practice

Determine whether the set of numbers in each table is proportional. If the relationship is proportional, identify the constant of proportionality. Explain your reasoning. (Examples 1 and 2)

1.

Blue Paint (quarts)	1	2	3	4
Yellow Paint (quarts)	5	6	7	8

2.

Ice Tea Mix (cups)	1	2	3	4
Sugar (cups)	2	4	6	8

3. The cost of 13 gallons of gasoline is $41.47. Find the constant of proportionality. Then write an equation relating cost to the number of gallons of gasoline. How much does 18.5 gallons of gasoline cost? (Example 3)

Independent Practice

Go online for Step-by-Step Solutions

Determine whether the set of numbers in each table is proportional. If the relationship is proportional, identify the constant of proportionality. Explain your reasoning. (Examples 1 and 2)

4.

Cans of Concentrate	1	2	3	4
Cans of Water	4	8	12	16

5.

Shaded Squares	1	2	3	4
Total Squares	8	15	30	42

6.

Junk E-mails	10	20	30	40
Total E-mails	15	30	45	60

7.

Weeks	5	6	7	8
Days	35	42	49	56

8. Financial Literacy A store is having a sale where all jeans are $\frac{1}{4}$ off the regular price. Find the constant of proportionality, then write an equation relating the sale price to the regular price. How much would a pair of $29 jeans cost on sale? (Example 3)

9. Luke earned $54 after mowing 3 lawns. Find the constant of proportionality, and then write an equation comparing earnings to lawns mowed. How much would Luke earn after mowing 7 lawns? (Example 3)

Copy and complete each table. Determine whether the relationship is proportional. If so, identify the constant of proportionality.

10. Ms. Rollins had an end-of-year pizza party for the chess team. At the party, every 2 students had 5 slices of pizza.

Number of Students	2	4	6	8	10
Slices of Pizza	■	■	■	■	■

11. Admission to an amusement park is $4 plus $1.50 per ride.

Number of Rides	1	2	3	4	5
Cost ($)	■	■	■	■	■

12. It will cost $7 per person to hold a birthday party at the recreation center.

Number of Guests	6	7	8	9	10
Cost ($)	■	■	■	■	■

13 Eight hot dogs and ten hot dog buns come in separate packages.

a. Is the number of packages of hot dogs proportional to the number of hot dogs? If the relationship is proportional, identify the constant of proportionality. Explain your reasoning.

b. Suppose you buy the same number of packages of hot dogs and hot dog buns. Is the number of hot dogs proportional to the number of hot dog buns? If the relationship is proportional, identify the constant of proportionality. Explain your reasoning.

$\frac{8}{10} = \frac{4}{5}$

$\frac{16}{20} = \frac{8}{10} = \frac{4}{5}$

$\frac{24}{30} = \frac{8}{10} = \frac{4}{5}$

a.

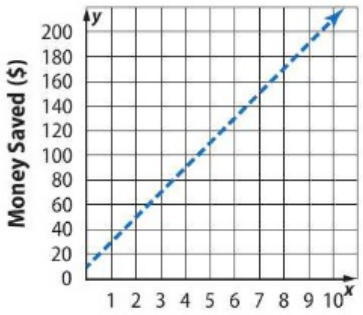
PKG HOT DOGS	HOT DOGS
1	8
2	16
3	24
10	80

b.

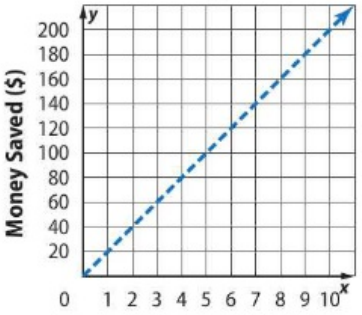
PKG HOT DOG	PKG BUNS
1	8
2	16
3	24
	10
	20
	30

14. CCSS **Multiple Representations** Suppose Isabel decides to save $20 each week for her family vacation. Her sister already has $10 and wants to save an additional $20 each week for the vacation. These situations are modeled in the graphs below.

Graph A



Graph B



a. **Table** Make a table showing the first six weeks of savings for each girl. Which situation is proportional? Explain your reasoning.

b. **Symbols** Write an equation to represent each situation.

c. **Graph** Compare and contrast the graphs.

H.O.T. Problems Higher Order Thinking

$\frac{79}{36} = \frac{2.194}{1}$

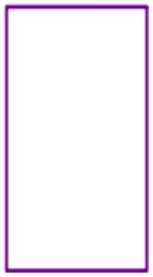
15. CCSS **Model with Mathematics** Give examples of two similar situations in which one is a proportional relationship and the second one is nonproportional. Identify the constant of proportionality. Then write equations that describe them.

16. CCSS **Justify Conclusions** A recipe for paper maché paste includes $\frac{1}{4}$ cup of flour for every cup of water. If there are 6 cups of flour, how many gallons of water are needed? Identify the constant of proportionality. Explain your reasoning.

17. CCSS **Persevere with Problems** Many objects, such as credit cards, are shaped like golden rectangles. A *golden rectangle* is a rectangle in which the ratio of the length to the width is approximately 1.618 to 1. This ratio is called the *golden ratio*.

36"

79"



a. Find three different objects that are close to a golden rectangle. Make a table to display the dimensions and the ratio found in each object.

b. Describe how each ratio compares to the golden ratio.

c. Use the Internet or another source to find three examples of where the golden rectangle is used in architecture.

$\frac{\text{LENGTH}}{\text{WIDTH}} = \frac{1.618}{1}$

GOLDEN RATIO

18. **Building on the Essential Question** This year Monica is 12 years old, and her little sister Patrice is 6 years old. Is Monica's age proportional to Patrice's age? If the relationship is proportional, identify the constant of proportionality. Explain your reasoning using a table of values.

Standardized Test Practice

19. A bicycle wheel makes 30 revolutions in 45 feet. Which of these represents an equivalent rate of bicycle wheel revolutions?

A 10 revolutions in 15 feet
B 60 revolutions in 100 feet
C 15 revolutions in 10 feet
D 100 revolutions in 60 feet

20. The cost of renting a boat for 4 hours is $51. If the cost of renting a boat is proportional to the number of rental hours, which of the following is *not* an equivalent rate?

F 6 hours for $76.50
G 3 hours for $38.25
H 7 hours for $89.25
J 5 hours for $63.00

21. The amount of sales tax paid on a purchase is proportional to the price of the item. Suppose the sales tax rate is 6.25%. If p is the price of the item and t is the sales tax, which equation represents this?

A $p = 0.0625t$
B $p = 6.25t$
C $t = 6.25p$
D $t = 0.0625p$

22. **Short Response** The prices of different-sized smoothies at an ice cream shop are shown below. Is the cost proportional to the size of the cone? If so, identify the constant of proportionality. Explain your reasoning.

Size (oz)	16	20	24
Cost ($)	3.25	3.75	4.25

Common Core Review

Complete each conversion. Round to the nearest hundredth, if necessary. 7.RP.1

23. 4 in. ≈ ■ cm
24. 5 L ≈ ■ gal
25. 1500 lb ≈ ■ kg
26. 14 yd ≈ ■ m

Express each rate as a unit rate. Round to the nearest tenth, if necessary. 7.RP.1

27. 140 miles on 6 gallons
28. 19 yards in 2.5 minutes
29. 236.7 miles in 4.5 days
30. 331.5 pages in 8.5 weeks
31. $5\frac{1}{2}$ feet in 3 minutes
32. 352 beats in $4\frac{2}{5}$ minutes

33. The table shows the record high and low temperatures for selected states. What is the difference between the highest and lowest temperatures for Kentucky? Massachusetts? 7.NS.1c

State	Lowest Temperature (°F)	Highest Temperature (°F)
Kentucky	−37	114
Massachusetts	−35	107

For Exercises 34–35, write an expression to represent each real-world situation. Then evaluate the expression and interpret the meaning of the solution. 7.NS.3

34. Nikos paid $22,500 for a new car. Over the next 3 years the value of his dropped $3750. How much did the value of the car drop each year on average?

35. The temperature dropped a total of 27°F over a 9-hour period. What was the mean hourly temperature drop?

ISG Interactive Study Guide
See page 107 for:
• Mid-Chapter Check

HEALTH CLUB

SIGN-UP FEE = $50

MONTHLY = $75

0 $50

1 $125 ($50 + $75) ONLY ONCE

$50 + 75x

x = MONTHS

5 $425

$50 + 75(5)

$50 + 375 = 425

$\frac{125}{1} \neq \frac{200}{2} \neq \frac{275}{3}$

PROPORTIONAL

MONTHLY = $80

0 $0

1 $80 80x

5 $400

$\frac{80}{1} = \frac{160}{2} = \frac{240}{3}$