Lesson 5-4
Converting Rates

Interactive Study Guide

See pages 103–104 for:
- Getting Started
- Real-World Link
- Notes

Essential Question

How can you identify and represent proportional relationships?

Common Core State Standards

Content Standards
7.RP.1, 7.RP.3

Mathematical Practices
1, 3, 4, 6, 7

Vocabulary

dimensional analysis

What You'll Learn

- Convert rates using dimensional analysis.
- Convert between systems of measurement.

Real-World Link

Sea Creature Leafy seadragons get their name from the leaflike appendages that camouflage them. They move very slowly using tiny fins on their necks and backs. You can use fractions involving units to calculate their average speed in inches per minute.

Dimensional Analysis

Dimensional analysis is the process of including units of measurement as factors when you compute. For example, you know that 1 hour = 60 minutes. You can write conversion factors $\frac{1 \text{ hour}}{60 \text{ minutes}}$ or $\frac{60 \text{ minutes}}{1 \text{ hour}}$. Each ratio is equivalent to 1 because the numerator and denominator represent the same amount.

Example 1

Tutor

Convert 100 miles per hour to miles per minute.

Step 1 You need to convert miles per hour to miles per minute. Choose a conversion factor that converts hours to minutes, with minutes in the denominator.

Convert miles per hour… … to miles per minute.

$$\frac{\text{miles}}{\cancel{\text{hour}}} \cdot \frac{\cancel{\text{hour}}}{\text{minute}} = \frac{\text{miles}}{\text{minute}}$$

So, use $\frac{1 \text{ h}}{60 \text{ min}}$. Conversion factor

Step 2 Multiply.

$$\frac{100 \text{ mi}}{1 \text{ h}} = \frac{100 \text{ mi}}{1 \text{ h}} \cdot \frac{1 \text{ h}}{60 \text{ min}}$$ Multiply by $\frac{1 \text{ h}}{60 \text{ min}}$.

$$= \frac{100 \text{ mi}}{\cancel{1 \text{ h}}_{1}} \cdot \frac{\overset{1}{\cancel{1 \text{ h}}}}{60 \text{ min}}$$ Divide out common units.

$$= \frac{100 \text{ mi}}{60 \text{ min}} \text{ or about } \frac{1.7 \text{ mi}}{1 \text{ min}}$$ Simplify.

So, 100 miles per hour is about 1.7 miles per minute.

Got It? Do this problem to find out.

1. **Financial Literacy** The average teenager spends $1742 per year on fashion-related items. How much is this per week?

Example 2

Tyree and three friends attend skydiving class before their first jump. The instructor tells them they will travel at about 176 feet per second. How many miles per hour is this?

Watch Out!
Make sure that you choose conversion factors that allow you to divide out the common units.

You need to convert feet per second to miles per hour.

Use 1 mile = 5280 feet and 1 hour = 3600 seconds.

$$\frac{176 \text{ ft}}{1 \text{ s}} = \frac{176 \text{ ft}}{1 \text{ s}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ s}}{1 \text{ h}}$$ Multiply by $\frac{1 \text{ mi}}{5280 \text{ ft}}$ and $\frac{3600 \text{ s}}{1 \text{ h}}$.

$$= \frac{176 \text{ ft}}{1 \text{ s}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ s}}{1 \text{ h}}$$ Divide the common factors and units.

$$= \frac{120 \text{ mi}}{1 \text{ h}}$$ Simplify.

So, 176 feet per second is equivalent to 120 miles per hour.

Got It? Do these problems to find out.

2a. The TGV is a high speed rail train in France. At top speed, it runs at an average of 320 kilometers per hour. How many meters per second is this?

2b. An adult cheetah can reach a speed of about 70 miles per hour. How fast is this in feet per second?

Key Concept Measurement Conversions

Length	
Customary to Metric	**Metric to Customary**
1 in. ≈ 2.540 cm	1 cm ≈ 0.394 in.
1 ft ≈ 0.305 m	1 m ≈ 3.279 ft
1 yd ≈ 0.914 m	1 m ≈ 1.094 yd
1 mi ≈ 1.609 km	1 km ≈ 0.621 mi
Capacity	
Customary to Metric	**Metric to Customary**
1 fl oz ≈ 29.574 mL	1 mL ≈ 0.034 fl oz
1 pt ≈ 0.473 L	1 L ≈ 2.114 pt
1 qt ≈ 0.946 L	1 L ≈ 1.057 qt
1 gal ≈ 3.785 L	1 L ≈ 0.264 gal
Mass or Weight	
Customary to Metric	**Metric to Customary**
1 oz ≈ 28.350 g	1 g ≈ 0.035 oz
1 lb ≈ 0.454 kg	1 kg ≈ 2.203 lb

The table above shows conversion factors between the Customary and Metric systems for units of length, capacity, and mass or weight.

You can also use dimensional analysis to convert between measurement systems. The two conversion factors $\frac{1 \text{ ft}}{0.305 \text{ m}}$ and $\frac{0.305 \text{ m}}{1 \text{ ft}}$ use the same conversion. Use the factor that will correctly divide out the appropriate common unit.

The TGV is a high speed rail train in France. At top speed, it runs at an average of 320 kilometers per hour. How many meters per second is this?

1 HR = 3,600 SEC

1 Km = 1,000 m

$$\frac{320\text{ Km}}{1\text{ HR}} \cdot \frac{1\text{ HR}}{3{,}600\text{ SEC}} = \frac{320\text{ Km}}{3{,}600\text{ SEC}} \cdot \frac{320{,}000\text{ m}}{320\text{ Km}} = \frac{320{,}000\text{ m}}{3{,}600\text{ SEC}} = \frac{88.89\text{ m}}{1\text{ SEC}}$$

$$36\overline{)3200} = 88.\overline{8}$$

288

320

288

32

2b. An adult cheetah can reach a speed of about 70 miles per hour. How fast is this in feet per second?

5,280(70)

$$\frac{70\text{ mi}}{1\text{ HR}} \cdot \frac{1\text{ HR}}{3{,}600\text{ SEC}} = \frac{70\text{ mi}}{3{,}600\text{ SEC}} \cdot \frac{369{,}600\text{ ft}}{70\text{ mi}} = \frac{369{,}600\text{ ft}}{3{,}600\text{ SEC}}$$

$$\frac{3696}{36}$$

$$36\overline{)3696} = 102.\overline{6}$$

$$\frac{102.67\text{ ft}}{1\text{ SEC}}$$

$$\frac{320\ \text{Km}}{1\ \text{HR}} \cdot \frac{320{,}000\ \text{m}}{320\ \text{Km}} = \frac{320{,}000\ \text{m}}{1\ \text{HR}} \cdot \frac{1\ \text{HR}}{3600\ \text{SEC}} = \frac{320{,}000\ \text{m}}{3{,}600\ \text{SEC}} \div \frac{3{,}600}{3{,}600} = \frac{88.\overline{8}\ \text{m}}{1\ \text{SEC}}$$

$320\ \text{Km} = 320{,}000\ \text{m}$

$1\ \text{Km} = 1000\ \text{m}$

$1\ \text{HR} = 60\ \text{MIN}$

$1\ \text{MIN} = 60\ \text{SEC}$

$1\ \text{HR} = 3600\ \text{SEC}$

$$\frac{70\ \text{mi}}{1\ \text{HR}} \cdot \frac{369600\ \text{ft}}{70\ \text{mi}} = \frac{369{,}600\ \text{ft}}{1\ \text{HR}} \cdot \frac{1\ \text{HR}}{3{,}600\ \text{SEC}} = \frac{369{,}600\ \text{ft}}{3{,}600\ \text{SEC}} \div \frac{3{,}600}{3{,}600} = \frac{102.\overline{6}\ \text{ft}}{1\ \text{SEC}}$$

$1\ \text{mi} = 5{,}280\ \text{ft}$

Example 3

Complete each conversion. Round to the nearest hundredth.

a. 12 centimeters to inches

Use 1 inch ≈ 2.54 centimeters.

$12 \text{ cm} \approx 12 \text{ cm} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}}$ Multiply by $\frac{1 \text{ in.}}{2.54 \text{ cm}}$.

$\approx 12 \text{ cm} \cdot \frac{1 \text{ in.}}{2.54 \text{ cm}}$ Divide out common units, leaving the desired unit, inch.

$\approx \frac{12 \text{ in.}}{2.54}$ or 4.72 in. Simplify.

So, 12 centimeters is approximately 4.72 inches.

b. 4 quarts to liters

Use 1 quart ≈ 0.946 liter.

$4 \text{ qt} \approx 4 \text{ qt} \cdot \frac{0.946 \text{ L}}{1 \text{ qt}}$ Multiply by $\frac{0.946 \text{ L}}{1 \text{ qt}}$.

$\approx 4 \text{ qt} \cdot \frac{0.946 \text{ L}}{1 \text{ qt}}$ Divide out common units, leaving the desired unit, quart.

$\approx 4 \cdot 0.946 \text{ L}$ or 3.78 L Simplify.

So, 4 quarts is approximately 3.78 liters.

Got It? Do these problems to find out.

3a. 6 mi ≈ ■ km **3b.** 12 oz ≈ ■ g **3c.** 11 yd ≈ ■ m

Example 4

Dimensions
When converting between systems, ask yourself how many dimensions are involved. In Example 4, there are two dimensions involved, length and time.

At top speed, a giant tortoise can travel about 900 feet per hour. How many centimeters per second can a giant tortoise travel at top speed?

To convert feet to centimeters, use 1 foot = 12 inches and 1 inch ≈ 2.54 centimeters.

To convert hours to seconds, use 1 hour = 60 minutes and 1 minute = 60 seconds.

$\frac{900 \text{ ft}}{1 \text{ h}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}}$

$= \frac{900 \text{ ft}}{1 \text{ h}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}}$ Divide out common units.

$= \frac{27{,}432 \text{ cm}}{3600 \text{ s}}$ Multiply.

$= \frac{7.62 \text{ cm}}{1 \text{ s}}$ Divide.

At top speed, a giant tortoise will travel 7.62 centimeters per second.

Got It? Do this problem to find out.

4. At a recent Winter Olympics, USA short track speed skater Apolo Ohno won a gold medal by skating about 12 meters per second. Rounded to the nearest hundredth, how many miles per hour is this?

Guided Practice

1. In Brazil, about 20 acres of rain forest are destroyed each minute. At this rate, how much rain forest is destroyed per day? (Example 1)

2. Lexi can paint 5 yards of fencing in one hour. At this rate, how many inches does she paint per minute? (Example 2)

Complete each conversion. Round to the nearest hundredth, if necessary. (Example 3)

3. 8 in. ≈ ■ cm
4. 5 L ≈ ■ gal
5. 15 oz ≈ ■ g
6. 24 cm ≈ ■ in.
7. 9 pt ≈ ■ L
8. 3 m ≈ ■ ft

9. An elephant can eat up to 440 pounds of vegetation every day. How many grams per minute is this? Round to the nearest hundredth. (Example 4)

Independent Practice

Go online for Step-by-Step Solutions

10. A candy company can produce 4800 sour lemon candies per minute. How many candies can they produce each hour? (Example 1)

11. In a recent year, 51.9 billion aluminum cans were recycled. About how many cans per week is this? (Example 1)

12. The average American student spends almost 1500 hours per year watching television. To the nearest hundredth, how many minutes per day is this? (Example 2)

13. A thrill ride at an amusement park travels 55 miles per hour. To the nearest hundredth, how many feet per second is this? (Example 2)

Complete each conversion. Round to the nearest hundredth, if necessary. (Example 3)

14. 4 L ≈ ■ qt
15. 16 in. ≈ ■ cm
16. 13 m ≈ ■ ft
17. 8 yd ≈ ■ m
18. 18 lb ≈ ■ kg
19. 7 L ≈ ■ gal
20. 1500 g ≈ ■ oz
21. 15 ft ≈ ■ m
22. 28 fl oz ≈ ■ mL

23. STEM The velocity of sound through wood at 0° Celsius is 1454 meters per second. How many miles is this per hour? Round to the nearest hundredth. (Example 4)

24. A certain car in Canada can travel 15 kilometers per 1 liter of gasoline. How many miles per gallon is this? Round to the nearest hundredth. (Example 4)

Complete each conversion. Round to the nearest hundredth, if necessary.

25. 8 in. ≈ ■ mm
26. 16 L ≈ ■ c
27. 2 km ≈ ■ yd
28. 250 fl oz ≈ ■ L
29. 2750 g ≈ ■ lb
30. 5 gal ≈ ■ mL

31. Crystal's times for each portion of a triathlon are shown in the table. Round to the nearest hundredth.

	Swim	Bike	Run
Distance (km)	1.5	40	10
Time (min)	40	86	64

   a. How many meters per second did she run?

   b. What was her speed in miles per hour for the aquabike portion (swimming and biking)?

Creative Crop/Digital Vision/Getty Images

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27. $\frac{2750\text{ g}}{1000} = \text{x lb}$

$2750\text{ g} = 2.75\text{ kg}$

2.75

$1\text{ kg} = 2.203\text{ lb}$

$2.75\text{ kg} = 6.06\text{ lb}$

23. $\frac{1454\text{ m}}{1\text{ SEC}} \cdot \frac{60\text{ SEC}}{1\text{ MIN}} \cdot \frac{60\text{ MIN}}{1\text{ HR}} = \frac{5,234,400\text{ m}}{1\text{ HR}} \cdot \frac{1\text{ mi}}{1609.34\text{ m}} = \frac{5,234,400}{1609.34} = \frac{3,252.5\text{ mi}}{1\text{ HR}}$

$\frac{1454\text{ m}}{1\text{ SEC}} \cdot \frac{0.90148\text{ mi}}{1454\text{ m}} = \frac{0.90148\text{ mi}}{1\text{ SEC}} \cdot \frac{1\text{ SEC}}{0.000278\text{ HR}} = \frac{0.90148\text{ mi}}{0.000278\text{ HR}} \cdot \frac{3600}{3600} = \frac{3245.33\text{ mi}}{1\text{ HR}}$

$1\text{ SEC} = 0.000278\text{ HR}$

$\frac{1}{3600} = 0.000278$

$1\text{ mi} = 1609.34\text{ m}$

$\frac{1\text{ km}}{1000} = \frac{0.62\text{ m}}{1000}$

$1\text{ m} = 0.00062\text{ mi}$

Order each group of rates from least to greatest.

32. 100 oz/min, 2500 g/min, 10 lb/min

33. 500 m/h, 7 yd/min, 6 in./s

34. 32 mi/gal, 15 m/mL, 6600 yd/qt

35. 500 kg/h, 5 oz/s, 18 lb/min

36. STEM The sprinkler system in the Willis Tower pumps up to 1500 gallons of water per minute. How many liters of water can the system pump in $\frac{1}{4}$ minute? Round to the nearest hundredth.

37 The average American consumes 20 gallons of ice cream in one year. At this rate, how many liters of ice cream will 50 Americans consume in one week? Round to the nearest hundredth.

Replace each ● with <, >, or = to make a true sentence.

38. 10 m ● 390 in. **39.** 520 oz ● 15 kg **40.** 14 pt ● 6622 mL

Financial Literacy Use dimensional analysis and data in the table to make each conversion. Round to the nearest hundredth, if necessary.

41. 150 dollars to euros

42. 275 dollars to pounds

43. 570 yuan to dollars $89.41

44. 500 pesos to dollars

Exchange Rates Per 1 U.S. Dollar		
Country	Currency	Rate
European Union	euro	0.790
Mexico	peso	13.444
China	yuan	6.375
United Kingdom	pound	0.637

$\frac{\text{DOLLARS}}{\text{YUAN}} = \frac{1}{6.375}$

$\frac{1}{6.375} = \frac{x}{570}$

$\frac{6.375x}{6.375} = \frac{570}{6.375}$

$x = 89.41$

45. CCSS **Model with Mathematics** Write and solve a real-world problem in which dimensional analysis is used to convert square feet to square yards.
Hint: 1 square yard = 9 square feet

H.O.T. Problems Higher Order Thinking

46. CCSS **Be Precise** Give two examples of different measurements that are equivalent to 10 centimeters per second.

47. CCSS **Which One Doesn't Belong?** Select the rate that does not have the same value as the other three. Explain your reasoning.

60 mi/h | 88 ft/s | 500 ft/min | 1440 mi/day

48. CCSS **Persevere with Problems** A recipe for fruit punch uses the ingredients shown in the table. About how many cups of each ingredient are needed? Round to the nearest tenth.

Fruit Punch	
900 mL	cranberry juice
700 mL	apple juice
300 mL	pineapple juice
150 mL	lemon juice
900 mL	club soda

49. CCSS **Identify Structure** What property of multiplication allows you to multiply a rate by a conversion factor without changing its value? Explain.

50. **Building on the Essential Question** Explain how you would convert 10 miles per hour to meters per second.

Standardized Test Practice

51. A speed of 55 miles per hour is the same rate as which of the following?

A 34 kilometers per hour
B 50 kilometers per hour
C 88 kilometers per hour
D 98 kilometers per hour

52. A piece of notebook paper measures $8\frac{1}{2}$ inches by 11 inches. Which of the following metric approximations is the same?

F 2 m by 2.8 m
G 3 cm by 4 cm
H 22 cm by 28 cm
J 30 m by 40 m

53. A car's mileage is registered at 29,345.5 miles. The driver sees a sign that warns of road work in 1000 feet. What will be the car's mileage when the road work begins?

A 29,345.7
B 29,345.9
C 29,356.2
D 29,356.5

54. **Short Response** Convert 565 miles per hour to feet per second. Show the procedure you used.

Common Core Review

Express each rate as a unit rate. Round to the nearest tenth or to the nearest cent, if necessary. 7.RP.1

55. $183 for 4 concert tickets

56. 100 feet in 14.5 seconds

57. 254.1 miles on 10.5 gallons

58. 9 inches of snow in 12 hours

59. **Financial Literacy** Mrs. Gallagher wants to buy the package of soda that is less expensive per can. Which pack of sodas shown should she buy? Explain your reasoning. 7.RP.1

Express each ratio as a fraction in simplest form. 7.RP.1

60. 12 cars out of 30 vehicles

61. 5 cups to 5 quarts

62. 15 soccer balls out of 35 balls

63. 8 pencils to 20 crayons

Simplify each expression. 6.EE.3

64. $(x - 3) + 2$

65. $(8 \cdot y) \cdot (-4)$

66. $25 + (d - 8)$

67. $9(5m)$

68. $(x + 1) - 9$

69. $5(3 \cdot r)$

70. Clive is making hamburgers for a cookout. How many $\frac{1}{4}$-pound hamburgers can he make from $2\frac{3}{4}$ pounds of ground beef? 7.NS.3

Find each product or quotient. 7.NS.2

71. $-12 \cdot (-10)$

72. $-18 \div 3$

73. $9 \cdot (-14)$

74. $54 \div (-6)$

75. $-14 \cdot 2$

76. $-72 \div (-4)$

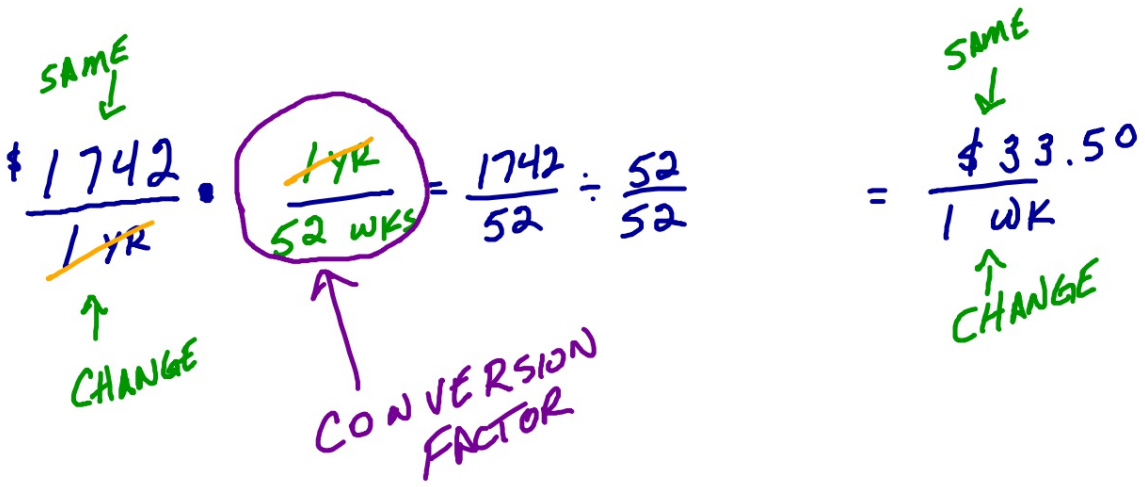
Write each fraction as a decimal. 7.NS.2d

77. $\frac{4}{5}$

78. $\frac{3}{4}$

79. $\frac{3}{8}$

80. $\frac{9}{25}$


SAME
$1742 / 1 YR
CHANGE
1 YR / 52 WKS
CONVERSION FACTOR
= 1742 ÷ 52 / 52
SAME
= $33.50 / 1 WK
CHANGE