Lesson 7-5

Factoring Linear Expressions

Essential Question

Why are algebraic rules useful?

Common Core State Standards

Content Standards
7.EE.1

Mathematical Practices
1, 2, 3, 4, 7

Vocabulary

factor
factored form

What You'll Learn

- Find the greatest common factor of two monomials.
- Use properties to factor linear expressions.

Real-World Link

Marching Band Band directors create geometrical formations that are eye-catching and exciting but still follow the rhythm and feel of the music. Graph paper is used to draw formations, with different colored ink representing different sections of the band.

[handwritten: COMMON FACTOR]
$$\frac{15}{20} = \frac{5 \cdot 3}{5 \cdot 4} = \frac{3}{4}$$

Find the GCF of Monomials

To **factor** a number means to write it as a product of its factors. A monomial can be factored using the same method you would use to factor a number. The greatest common factor (GCF) of two monomials is the greatest monomial that is a factor of both.

Example 1

Find the GCF of each pair of monomials.

a. $4x$, $12x$

$4x = 2 \cdot 2 \cdot x$ Write the prime factorization of $4x$ and $12x$.
$12x = 2 \cdot 2 \cdot 3 \cdot x$ Circle the common factors.

The GCF of $4x$ and $12x$ is $2 \cdot 2 \cdot x$ or $4x$.

[handwritten: $\frac{4x}{12x} = \frac{4x}{4 \cdot 3x} = \frac{1}{3}$]

b. $18a$, $20ab$

$18a = 2 \cdot 3 \cdot 3 \cdot a$ Write the prime factorization of $18a$ and $20ab$.
$20ab = 2 \cdot 2 \cdot 5 \cdot a \cdot b$ Circle the common factors.

The GCF of $18a$ and $20ab$ is $2 \cdot a$ or $2a$.

[handwritten: GCF; 4, 12 ← 4 GCF; 4x, 12x ← 4x GCF]

Got It? Do these problems to find out.

Find the GCF of each pair of monomials.

1a. 12, $28c$ 1b. $25x$, $15xy$ 1c. $42mn$, $14mn$

[handwritten answers: 1a. 4; 1b. 5x; 1c. 42mn = 3·14·m·n, 14mn = 14·m·n]

[handwritten margin: 12 : 3, 4; 28c : 4, 7, c]

Factor Linear Expressions

You can use the Distributive Property and the work backward strategy to express an algebraic expression as a product of its factors. An algebraic expression is in **factored form** when it is expressed as the product of its factors.

$$8x + 4 = 4(2x) + 4(1) \qquad \text{The GCF of } 8x \text{ and } 4 \text{ is } 4.$$
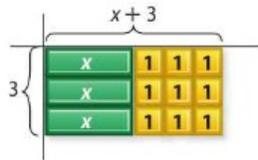$$= 4(2x + 1) \qquad \text{Distributive Property}$$

Example 2

Tutor

Factor each expression.

a. $3x + 9$

Method 1 Use a model.

Model $3x + 9$.

$x + 3$

Arrange the tiles into equal rows and columns.
The rectangle has a width of three 1-tiles, or 3, and
a length of one x-tile and three 1-tiles, or $x + 3$.

So, $3x + 9 = 3(x + 3)$.

Method 2 Use the GCF.

$3x = \boxed{3} \cdot x \qquad$ Write the prime factorization of $3x$ and 9.
$9 = \boxed{3} \cdot 3 \qquad$ Circle the common factors.

The GCF of $3x$ and 9 is 3. Write each term as a product of the GCF and its remaining factors.

$3x + 9 = \mathbf{3}(x) + \mathbf{3}(3)$
$= \mathbf{3}(x + 3) \qquad$ Distributive Property

So, $3x + 9 = 3(x + 3)$.

b. $12x + 7$

Find the GCF of $12x$ and 7.

$12x = 2 \cdot 2 \cdot 3 \cdot x$
$7 = 1 \cdot 7$

There are no common factors, so $12x + 7$ *cannot be factored.*

> **Factoring Expressions**
> Use algebra tiles to model the expression in Example 2b. Since you cannot rearrange the tiles to make a rectangle, the expression cannot be factored.

Got It? Do these problems to find out.

Factor each expression. If the expression cannot be factored, write *cannot be factored.* Use algebra tiles if needed.

2a. $4x + 28$

2b. $3 + 33x$

2c. $4x + 35$

Handwritten annotations:
$15x + 20 \quad GCF\ 5$
$5(3x + 4)$

$3x + 9 \quad GCF\ 3$
$3(x + 3)$
$3 \cdot \square = 3x \qquad 3 \cdot \square = 9$

RELATIVELY PRIME
$4x = 2 \cdot 2 \cdot x$
$35 = 5 \cdot 7$
$\frac{4}{9} = \frac{4}{9}$

2a. $4(x + 7)$
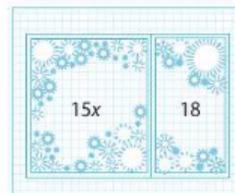2b. $3(1 + 11x)$
2c. CANNOT BE FACTORED

Example 3

The garden at the right has a total area of $(15x + 18)$ square feet. Find possible dimensions of the garden.

Factor $15x + 18$.

$15x = 3 \cdot 5 \cdot x$ Write the prime factorization of $15x$ and 18.

$18 = 2 \cdot 3 \cdot 3$ Circle the common factors.

The GCF of $15x$ and 18 is 3. Write each term as a product of the GCF and its remaining factors.

$15x + 18 = \mathbf{3}(5x) + \mathbf{3}(6)$

$ = \mathbf{3}(5x + 6)$ Distributive Property

So, the dimensions of the garden are 3 feet and $(5x + 6)$ feet.

Check   Find the product of 3 and $5x + 6$. $3(5x + 6) = 15x + 18$ ✓

Got It? Do these problems to find out.

3a. Financial Literacy The Reyes family has saved \$480 as a down payment for a new television. If x is the monthly payment for one year, the expression \$12x + \$480 represents the total cost of the television. Factor \$12x + \$480.

3b. Jesse wants to put down \$100 toward a new computer and will pay it off in six months. If y is the monthly payment, what expression represents the total price?

6 PAYMENTS OF Y AND \$100   *6y +100*

Guided Practice

Find the GCF of each pair of monomials. (Example 1)

1. $32x, 18$
2. $15y, 25$
3. $45a, 20a$
4. $16b, 12b$
5. $42s, 28s$
6. $56g, 84gh$
7. $27s, 54st$
8. $18cd, 30cd$
9. $22mn, 11kmn$

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed. (Example 2)

10. $36x + 24$
11. $6 + 3x$
12. $4x + 9$
13. $13x + 21$
14. $2x - 4$
15. $14x - 16$
16. $12 + 18x$
17. $24 + 32x$
18. $15x + 8$

19. Mr. Phen's monthly income can be represented by the expression $25x + 120$, where x is the number of hours worked. Factor the expression $25x + 120$. (Example 3)

20. The area of a high school basketball court is $(50x - 300)$ square feet. Factor $50x - 300$ to find possible dimensions of the basketball court. (Example 3)

Independent Practice

eHelp

Find the GCF of each pair of monomials. (Example 1)

21. $24, 48m$

22. $63p, 84$

23. $40x, 60x$

24. $32a, 48b$

25. $30rs, 42rs$

26. $54gh, 72g$

27. $36k, 144km$

28. $60jk, 45jkm$

29. $100xy, 75xyz$

(handwritten left margin)
24   48m
6·4   6·8
24·1 24·2

(handwritten right margin)
$(4x-8)$
$4: 1, 2, 4)$
$8: 1, 2, 4, 8)$

Factor each expression. If the expression cannot be factored, write *cannot be factored*. Use algebra tiles if needed. (Example 2)

30. $3x + 9$

31. $5x + 5$

32. $10x - 35$

33. $2x - 15$ *(handwritten)* 0.5 | $4x-8$ | w   L

34. $4x - 7$

35. $32 + 24x$

36. $12 + 30x$

37. $18x + 6$

38. $30x - 40$

(handwritten) $8x - 16$

(handwritten) $4x - 8$ | $4x-8$ | w | L

(handwritten, top right)
$2x - 4$
2 | $4x-8$ | w   L

(handwritten)
$x - 2$
4 | $4x-8$ | w   L

39. The area of a rectangle is $(4x - 8)$ square units. Factor $4x - 8$ to find possible dimensions of the rectangle. (Example 3)

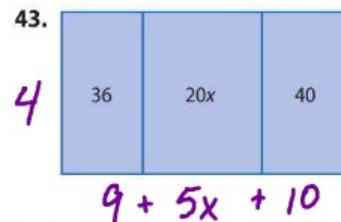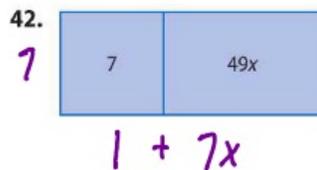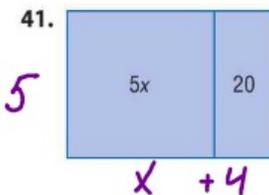40. James has $120 in his savings account and plans to save $x each month for 6 months. The expression $6x + $120 represents the total amount in the account after 6 months. Factor the expression $6x + 120$. (Example 3)
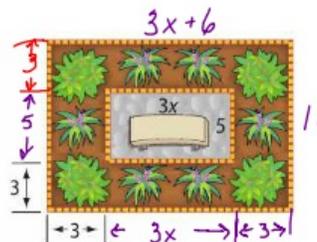
(handwritten)
$10x + 38$
$2(5x + 19)$

43. $20x + 76$
$2(10x + 38)$

Write an expression in factored form to represent the total area of each rectangle.

41.
(handwritten) 5

| 5x | 20 |

(handwritten) x $+4$

42.
(handwritten) 7

| 7 | 49x |

(handwritten) $1 + 7x$

43.
(handwritten) 4

| 36 | 20x | 40 |

(handwritten) $9 + 5x + 10$

44. A square scrapbooking page has a perimeter of $(8x + 20)$ inches. What is the length of one side of the page?

45. Six friends visited a museum to see the new holograms exhibit. The group paid for admission to the museum and $12 for parking. The total cost of the visit can be represented by the expression $6x + $12. What was the cost of the visit for one person?   *(handwritten)* $5 \angle$.

46. **CCSS Reason Abstractly** The diagram represents a flower border that is 3 feet wide surrounding a rectangular sitting area. Write an expression in factored form that represents the area of the flower border.

(handwritten)
$11(3x+6) = 33x + 66$
$-[5(3x)] = [15x + 0]$
$18x + 66$

$90x - 15 \Rightarrow 15(6x-1)$

(handwritten near diagram)
$3x + 6$
3x
5   11
3
$\mid$←3→$\mid$← 3x →$\mid$←3→$\mid$

(handwritten right)
$-\frac{5}{12}$
$\frac{\$6x}{6} = \$x$
$\$42 + \$12$
$-\$12$
$\frac{\$42}{6} = \$7$

Write an expression in factored form that is equivalent to the given expression.

47. $\frac{1}{2}x + 4 =$ *(handwritten)* $\frac{1}{2}(x+8)$

48. $\frac{2}{3}x + 6$ *(handwritten)* $\frac{2}{3}(x+9)$ $\frac{6}{\frac{2}{3}} = \frac{6}{1} \cdot \frac{2}{3} = \frac{6 \cdot 3}{1 \cdot 2} = \frac{18}{2} = 9$

49. $\frac{3}{4}x - 24$ *(handwritten)* $\frac{3}{4}(x - 32)$

$\frac{24}{\frac{3}{4}} = \frac{24 \cdot 4}{3} = \frac{32}{1} = 32$

50. $\frac{5}{6}x - 30$ *(handwritten)* $\frac{5}{6}(x - 36)$

$\frac{30}{\frac{5}{6}} = \frac{30}{1} \cdot \frac{6}{5} = \frac{36}{1}$

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39.

$$4\overline{)4x+8} \quad (x+2)$$

$$4(x+2)$$

$$4\overline{)32} \quad (8)$$

$$4(8)$$

$$2\overline{)4x+8} \quad (2x+4)$$

$$2(2x+4)$$

$$1\overline{)4x+8} \quad (4x+8)$$

$$1(4x+8)$$

$$\tfrac{1}{2}\overline{)4x+8} \quad (8x+16)$$

$$\tfrac{1}{2}(8x+16)$$

$$2\overline{)4x-8} \quad (2x-4)$$

$$2(2x-4)$$

$$4\overline{)4x-8} \quad (x-2)$$

$$4(x-2)$$

$$x > 2$$

$$\frac{4}{10} = \frac{2 \cdot \boxed{2}}{5 \cdot \boxed{2}} = \frac{2}{5}$$

$$\uparrow$$
GCF

$$\frac{4x}{10xy} =$$

$$\frac{\boxed{2} \cdot 2 \cdot \boxed{x}}{\boxed{2} \cdot 5 \cdot \boxed{x} \cdot y} = \frac{2}{5y}$$

GCF $2x$

$$2x - 4$$

2 $A = 4x - 8$

$$8x - 16$$

$\frac{1}{2}$ $4x - 8$

$$x - 2$$

4 $A = 4x - 8$

🔥 H.O.T. Problems Higher Order Thinking

51. **CCSS Identify Structure** Write two monomials whose greatest common factor is $4m$.

52. **CCSS Find the Error** Enrique is factoring $90x - 15$. Find his mistake and correct it.

$$90x - 15 = 15(6x)$$
$$= 90x$$

53. **🅮 Building on the Essential Question** Explain how the GCF is used to factor an expression. Use the term *Distributive Property* in your response.

✏️ Standardized Test Practice

54. **Short Response** Factor the expression $40x + 15$.

55. Which of the following expressions cannot be factored?

   A $6 + 3x$
   B $7x + 3$
   C $15x + 10$
   D $30x + 40$

56. The Venn diagram shows the factors of 12 and $18x$.
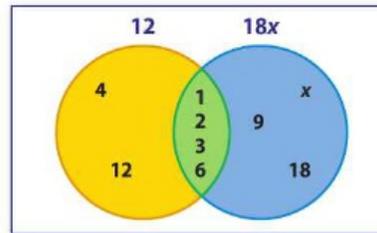
   What is the greatest common factor of the two monomials?

   F 2   **H** 6
   G 3   **J** 36

🅲🅲🅢🅢 Common Core Review

Find each product or quotient. 8.EE.1

57. $2^4 \cdot 2^6$

58. $\dfrac{a^3}{a^{-3}}$

59. $4x^{-2} \cdot 3x^9$

60. $\dfrac{c^5}{c^9}$

61. $(-4)s^{-8} \cdot (-4)s^7$

62. $\dfrac{12y^8}{6y^{10}}$

63. Tionne can ride 6 miles on her bike in one hour. If she rode for 1.5 hours on Saturday and 2 hours on Sunday, use mental math to find the total distance she rode that weekend. Justify your answer by using the Distributive Property. 7.NS.2c

64. A commission is a fee paid to a salesperson based on a percent of sales. Suppose a real estate agent earns a 3% commission. What commission would be earned for selling a house for $230,000? 7.RP.3

Add or subtract. 7.EE.1

65. $(-4x + 7) + (5x - 9)$

66. $(4.3x - 2) - (2.2x - 4)$

67. $\left(-\dfrac{5}{8}x + 3\right) + \left(\dfrac{3}{4}x - 8\right)$

68. $(6x - 4) - (6x + 1)$