



Lesson 8-7

Solving Inequalities



Interactive Study Guide

See pages 183–184 for:

- Getting Started
- Real-World Link
- Notes



Essential Question


How are equations and inequalities used to describe and solve multi-step problems?



Common Core State Standards

Content Standards
7.EE.4, 7.EE.4b

Mathematical Practices
1, 3, 4

What You'll Learn 

- Solve inequalities by using the Addition and Subtraction Properties of Inequality.
- Solve inequalities by multiplying or dividing by a positive or negative number.

Real-World Link 

Pets Did you know that 39 percent of U.S. households own at least one dog? The amount of food that you feed your dog should be based on the dog's weight. For example, an adult dog that weighs 10 pounds or less needs about $\frac{3}{4}$ cup of food per day. A dog that weighs more than 30 pounds needs 2 to 4 cups of food each day.



Key Concept Addition and Subtraction Properties

Words When you add or subtract the same number from each side of an inequality, the inequality remains true.

Symbols For all numbers a , b , and c ,

1. If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
2. If $a > b$, then $a + c > b + c$ and $a - c > b - c$.

Examples

$5 < 9$	$11 > 6$
$5 + 4 < 9 + 4$	$11 - 3 > 6 - 3$
$9 < 13$	$8 > 3$

The above properties are useful for solving inequalities. Note that these properties are also true for $a \leq b$ and $a \geq b$.

Example 1



Solve $x + 5 > 12$. Check your solution.

$$\begin{array}{r} x + 5 > 12 \\ -5 \quad -5 \\ \hline x > 7 \end{array}$$

Write the inequality.
Subtraction Property of Inequality
Simplify.

Check

$x + 5 > 12$	Write the inequality.
$9 + 5 > 12$	Replace x with any number greater than 7.
$14 > 12$ ✓	The statement is true.

Got It? Do these problems to find out.

1a. $y + 10 < 3$

1b. $x + 7 \geq 10$

Equations and Inequalities

In an equation, if $a = b$, then $b = a$. In an inequality, if $a < b$, then $b > a$.

Example: $7 = 2 + 5$
 $2 + 5 = 7$
 $2 + 8 > 7$
 but $7 < 2 + 8$

When graphing inequalities, it is often easier to visualize the solution when the variable is on the left side of the inequality symbol.

Example 2

Solve $3 \leq b - 1\frac{1}{3}$. Graph the solution on a number line.

$$3 \leq b - 1\frac{1}{3}$$

Write the inequality.

$$3 + 1\frac{1}{3} \leq b - 1\frac{1}{3} + 1\frac{1}{3}$$

Addition Property of Inequality

$$4\frac{1}{3} \leq b \text{ or } b \geq 4\frac{1}{3}$$

Simplify.

The solution is $b \geq 4\frac{1}{3}$.

Check $3 \leq b - \frac{1}{3}$

Write the inequality.

$$3 \stackrel{?}{\geq} 4\frac{1}{3} - 1\frac{1}{3}$$

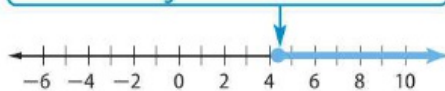
Replace b with $4\frac{1}{3}$.

$$3 \leq 3 \checkmark$$

The statement is true.

Graph the solution.

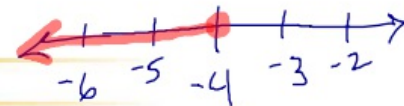
Since the inequality symbol is \geq , draw a closed dot at $4\frac{1}{3}$ with an arrow to the right.



2a $3 \geq g + 7$
 $-7 \quad -7$

$$-4 \geq g$$

$$g \leq -4$$



Got It? Do these problems to find out.

Solve each equation. Graph the solution on a number line.

2a. $3 \geq g + 7$

2b. $b + \frac{5}{7} > 2$

Positive Number

The statement $c > 0$ means that c is a positive number.

Key Concept Multiplication and Division Properties

Words When you multiply or divide each side of an inequality by the same *positive* number, the inequality remains true.

Symbols For all numbers a , b , and c , where $c > 0$,

1. If $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

2. If $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Examples $-6 < 10$ $20 > 16$

$$-6 \cdot 2 < 10 \cdot 2 \quad \frac{20}{4} > \frac{16}{4}$$

$$-12 < 20 \quad 5 > 4$$

These properties are also true for $a \leq b$ and $a \geq b$.

Some inequalities, like $4x > 8$, are solved by multiplication or division. You can multiply or divide each side of an inequality by a positive number and the inequality is still true.

$$2(4) > (3)^2$$

$$8 > 6$$

$$-2(4) > (3)(-2)$$

$$-8 < -6$$

$$\frac{-8}{2} < \frac{-6}{2}$$

$$-4 < -3$$

$$\frac{-8}{-2} < \frac{-6}{-2}$$

$$4 > 3$$

$$\boxed{5} x = 5x$$

$$\boxed{-\frac{1}{3}} x = \frac{x}{-3}$$

$$-5y - 3 > 8$$

$$+3 \quad +3$$

$$\frac{-5y}{-5} > \frac{11}{-5}$$

$$y < -\frac{11}{5}$$



$$\frac{x}{-3} - 5 > 1$$

$$+5 \quad +5$$

$$-3\left(\frac{x}{-3}\right) > 6(-3)$$

$$x < -18$$



Example 3



Words to Symbols

Remember, *at most* translates to \leq , while *at least* translates to \geq .

Macy is making each of her 7 friends a bracelet. In addition to charms, she needs one package of wire for all of the bracelets. The wire is \$6. She does not want to spend more than \$45 on the bracelets. Find the maximum cost for each bracelet.

Since Macy wants to spend at most \$45, write and solve an inequality using the symbol \leq .

Words	Number of friends times cost of each bracelet plus \$6 must be less than or equal to \$45.
Variable	Let c = the cost of each bracelet.
Inequality	$7c + \$6 \leq \45

$7c + 6 \leq 45$	Write the inequality.	
$7c + 6 - 6 \leq 45 - 6$	Subtraction Property of Inequality	$\frac{3}{4}(14) \leq h - \frac{1}{2}$
$7c \leq 39$	Simplify.	$9 \leq h - \frac{1}{2}$
$\frac{7c}{7} \leq \frac{39}{7}$	Division Property of Inequality	$+\frac{1}{2} \quad +\frac{1}{2}$
$c \leq 5\frac{4}{7}$ or 5.571428	Simplify.	$9\frac{1}{2} \leq h$

Macy can spend no more than \$5.57 per bracelet.

Got It? Do these problems to find out. $(9)\frac{3}{4} = 6.75$

- 3a. Alfonso works for a lawn service company. It takes Alfonso $\frac{3}{4}$ hour to mow a lawn. If he works more than 8 hours, he gets a $\frac{1}{2}$ hour lunch. Write and solve an inequality to find the number of lawns he can mow if he works at least 14 hours.
- 3b. Keiko prepared 28 bags of granola to sell at a school fundraiser. She also received a \$10 donation. Write and solve an inequality to find the price she should charge for each bag of granola if she wants to raise at least \$115.

Key Concept Multiplication and Division Properties

Words	When you multiply or divide each side of an inequality by the same <i>negative</i> number, the inequality symbol must be reversed for the inequality to remain true.
Symbols	For all numbers a , b , and c , where $c < 0$, 1. If $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. 2. If $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
Examples	$-4 < 5 \quad 18 > -12$ $-4 \cdot (-3) > 5 \cdot (-3) \quad \frac{18}{-3} < \frac{-12}{-3}$ $12 > -15 \quad -6 < 4$

These properties are also true for $a \leq b$ and $a \geq b$.

Handwritten work for Example 3:

$$\frac{3}{4}(14) - \frac{1}{2} = h$$

$$\frac{3}{4}(14h) - \frac{1}{2} =$$

$$\frac{14h}{\frac{3}{4}} - \frac{1}{2} \leq h$$

Handwritten work for problem 3a:

$$\frac{3}{4}h - \frac{1}{2} \leq 14$$

$$+\frac{1}{2} \quad +\frac{1}{2}$$

$$\frac{4}{3}(\frac{3}{4}h) \leq (\frac{29}{2})\frac{4}{3}$$

$$h \leq 19\frac{1}{3}$$

Labels: 3a, 14 1/2 =, 19 1/3, 29/2, 3/4, 4/3

Negative Number
 The statement $c < 0$ means that c is a negative number.

Handwritten work for problem 3b:

$$\frac{58}{3} \cdot \frac{3}{4}$$

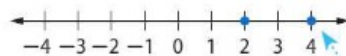
$$\frac{58}{4} = 14.5$$

Labels: TOO MUCH TIME



To understand the properties shown on the previous page, consider what happens when each side of an inequality is multiplied or divided by a negative number.

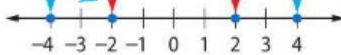
Graph 2 and 4 on a number line.



Since 4 is to the right of 2, $2 < 4$.

Now, multiply each number by -1 .

Since -2 is to the right of -4 , $-2 > -4$.



Notice that the numbers being compared switched positions as a result of being multiplied by a negative number. In other words, their order reversed. This suggests the properties shown on the previous page.



Example 4



Solve each inequality. Then graph the solution on a number line.

Checking Solutions

For Example 4a, try a number that is less than -9 to show it is *not* a solution.

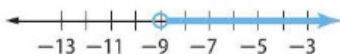
$$\begin{aligned} -5x &< 45 \\ -5(-10) &\stackrel{?}{<} 45 \\ 50 &\not< 45 \end{aligned}$$

a. $-5x < 45$

$$-5x < 45 \quad \text{Write the inequality.}$$

$$\frac{-5x}{-5} > \frac{45}{-5} \quad \text{Division Property of Inequality}$$

$$x > -9 \quad \text{Simplify.}$$



b. $\frac{b}{-8} - 2 \geq -8$

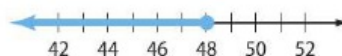
$$\frac{b}{-8} - 2 \geq -8 \quad \text{Write the inequality.}$$

$$\frac{b}{-8} - 2 + 2 \geq -8 + 2 \quad \text{Addition Property of Inequality}$$

$$\frac{b}{-8} \geq -6 \quad \text{Simplify.}$$

$$(-8)\frac{b}{-8} \leq -6(-8) \quad \text{Multiplication Property of Inequality}$$

$$b \leq 48 \quad \text{Simplify.}$$



Got It? Do these problems to find out.

4a. $-\frac{y}{4} < 3$

4b. $7 \geq -2f$

Guided Practice



Solve each inequality. Check your solutions. (Example 1)

1. $y + 7 \leq 12$

2. $b + 20 > -13$

3. $-7 < x + (-3)$

Solve each inequality. Graph each solution on a number line. (Examples 2 and 4)

4. $d - 9.3 \geq 12.5$

5. $3\frac{1}{5} > f - \frac{4}{5}$

6. $g - 22 \leq -40$

7. $-12 \geq -3q - 18$

8. $-8z \leq -24$

9. $18 > -\frac{2}{3}g$



10. Isabel earns \$50 plus \$2.50 for each table she cleans. Write and solve an inequality to find how many tables she must clean to earn at least \$120. (Example 3)

$$8. \frac{-8z}{-8} \leq \frac{-24}{-8}$$

$$z \leq 3$$

$$-8(4) \leq -24$$

$$-32 \leq -24$$

$$-8(2) \leq -24$$

$$-16 \leq -24$$

$$9. 18 > -\frac{2}{3}g$$

21 BUS THEM TO PUMPKIN PATCH

BUSSES HOLD 10 STUDENTS

HOW MANY BUSES?

Independent Practice

Go online for Step-by-Step Solutions



Solve each inequality. Check your solutions. (Examples 1 and 2)

11. $a + 18 < 40$

12. $h - 12 > 52$

13. $y - 4.2 \leq 6.5$

14. $g + 5.9 \geq 10$

15 $p - 14 > 12$

16. $x + 3.75 < 5$

17. $n - 0.1 \leq 1.4$

18. $7 > z + \frac{2}{3}$

19. $22 \geq c - 2.1$

20. $13 \geq 9 + b$

21. $14\frac{1}{2} < b - 1\frac{1}{4}$

22. $t + \frac{1}{5} < 2\frac{7}{10}$

23. Montel pays a \$2 entrance fee and \$0.75 every time he plays his favorite video game. If he has \$10, write and solve an inequality to find how many video games he can play. (Example 3)

24. The dance committee has \$75 to spend on centerpieces. They spent \$30 on flowers. If each vase costs \$3, write and solve an inequality to find how many vases they can buy. (Example 3)

Solve each inequality. Graph each solution on a number line. (Example 4)

25. $4x > -36$

26. $7y \leq -49$

27. $13 - 3n < -8$

28. $3b + 15 \leq 8b - 5$

29. $45 \leq -10r$

30. $15 \leq -\frac{k}{4} - 9$

31. $\frac{7}{2}y > 63$

32. $\frac{3}{4} \leq -\frac{5}{7}m$

33. $-\frac{3}{24}b \geq -\frac{1}{4}$

34. $-\frac{3}{4}d - 6 \geq 9$

35. $\frac{4}{9}c \leq -\frac{4}{5}$

36. $-\frac{3}{4} \geq -\frac{6}{10}a$

37 Khadijah has at most three hours to work on a math assignment and a history project. If the math assignment will take $\frac{3}{4}$ hour, how much time can Khadijah spend working on her history project?

38. The 2008 attendance at the Ohio State Fair was at least 16,700 less than the attendance in 2009. If the attendance in 2008 was 809,300, write and solve an inequality to find the 2009 attendance.

Solve each inequality. Check your solutions.

39. $a - 3.5 < \frac{2}{5}$

40. $b + 4\frac{1}{2} \geq 0.4$

41. $26 \leq 5 - c$

42. $12 - d > 40$

43. $4.5 \geq \frac{3}{2}r$

44. $f - 8 < -1.1$

45. $t - 6 \leq 2.5$

46. $-9.6 \geq \frac{1}{3}y$

47. $-\frac{7}{11} \geq -\frac{14}{33}g$

Write an inequality to represent each situation. Then solve each inequality.

48. Seven more than a number is at most 24.

49. The quotient of a number and -3 is greater than the quotient of 5 and 6.

50. 18 is at least the product of -6 and a number.

51. The difference of a number and 15 is no more than -8 .

52. Twelve less than a number is at most 20.

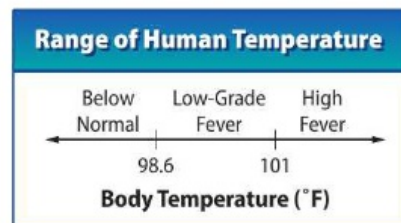
53. 28 is at least the product of 7 and a number.

54. **Financial Literacy** Brian is saving money to buy a new mountain bike. The bike that he likes costs \$375.95, and he has already saved \$285.50. Write and solve an inequality to find the amount he must still save.
55. **CCSS Multiple Representations** Consider the inequalities $b \geq 4$ and $b \leq 13$.
- Graph** Graph each inequality on the same number line.
 - Words** Do the solution sets of the two inequalities overlap? If so, what does this overlapping area represent?
 - Symbols** A *compound inequality* is an inequality that combines two inequalities. Write a compound inequality for the situation.
 - Graph** Look back at the graph of the solutions for both inequalities. Make another graph that shows only the solution of the compound inequality.

Graph each compound inequality on a number line.

56. $-3 < n < 5$ 57. $4 \geq m > -2$ 58. $8 \leq g < 14$

59. **STEM** Use the body temperature scale shown.
- Suppose Malia has a temperature of 99.2°F . Write and solve an inequality to find how much her temperature must increase before she is considered to have a high fever.
 - Hypothermia occurs when a person's body temperature falls below 95°F . Write and solve an inequality that describes how much lower the body temperature of a person with hypothermia will be than the body temperature of a person with a normal temperature of 98.6°F .



H.O.T. Problems Higher Order Thinking

60. **CCSS Persevere with Problems** Write an inequality for the following sentence.
The quotient of a number and -5 increased by 4 is at most 8.
 Name three numbers that are possible solutions. Explain.
61. **CCSS Model with Mathematics** Write a real-world problem involving an inequality and negative numbers where the inequality symbol would *not* be reversed when finding the solution.
62. **CCSS Persevere with Problems** Twenty more than half a number is at least 45. Find the least number that meets this condition.
63. **CCSS Use a Counterexample** Is the following statement *true* or *false*? If false, provide a counterexample.
 For all values of x , two times x is greater than x .
64. **CCSS Justify Conclusions** Is it *always*, *sometimes*, or *never* true that if $x \leq y$, then $y > x$? Explain your reasoning to a classmate.
65. **Building on the Essential Question** Explain how to solve inequalities that involve multiplication and division.



Standardized Test Practice

66. The length of the rectangle is greater than its width. Which inequality represents the possible values of x ?



- A** $x \leq 17$ **C** $x \geq 17$
B $x < 17$ **D** $x > 17$
67. If $n + 15 > 4$, then n could be which of the following values?
- F** -13 **H** -11
G -12 **J** -10
68. The solutions for which inequality are represented by the following graph?



A $\frac{x}{-3} < 5$ **C** $\frac{x}{3} > -5$

B $\frac{x}{-3} \leq 5$ **D** $\frac{x}{-3} \geq 5$

69. The product of a number n and four is at most thirty. Which inequality represents the possible values of n ?

F $n \leq 7\frac{1}{2}$ **H** $n \geq 120$

G $n \geq 7\frac{1}{2}$ **J** $n \leq 120$



Common Core Review

Write an inequality for each sentence. 7.EE.4

70. Leticia made at least \$45 babysitting last weekend.
71. Marc could pay no more than \$8500 for his car.
72. Adrienne needs an 86% or better to get a B in the class.
73. There are fewer than 625 students at Everett Middle School.
74. The car's tank contains no less than 8.5 gallons of gasoline.
75. Tyrell has more than 200 foreign coins in his collection.

Solve each equation. Check your solutions. 8.EE.7

76. $3x + 1 = 7$

77. $5x - 4 = 11$

78. $4h + 6 = 22$

79. $8n + 3 = -5$

80. $37 = 4d + 5$

81. $9 = 15 + 2p$

82. $4h + 5 = -6h - 19$

83. $7d - 13 = 17 + 3d$

84. $n - 14 = 3n$

85. $2g + 12 = 3g - 1$

86. $8y + 5 = 5y - 5 + 2y$

87. $4t = 2t - 26$

88. A standard showerhead uses about 6 gallons of water per minute. The table shows the relationship between time in minutes and the number of gallons of water used. 7.RP.2

- a. Given m , the number of minutes, write an equation that can be used to find g , the number of gallons used.
- b. How many minutes elapsed if 72 gallons of water were used?

Taking a Shower	
Time m (minutes)	Water Used g (gallons)
1	6
2	12
4	24
7	42