Please work these problems in your spiral.

$3(1+2f)-5=6f-2$

$3+6f-5=6f-2$

$6f-2=6f-2$

$+2 \quad +2$

$\frac{6f}{6}=\frac{6f}{6}$

$f=f$

INFINITE SOLUTIONS

$6(-2z+5)<-19z+16$

$-12z+30<-19z+16$

$-16 \quad -16$

$-12z+14<-19z$

$+12z \quad +12z$

$\frac{14}{-7}<\frac{-7z}{-7}$

$-2>z$

$z<-2$

Lesson 8-8
Solving Multi-Step Equations and Inequalities

Interactive Study Guide
See pages 185–186 for:
- Getting Started
- Real-World Link
- Notes

Essential Question
How are equations and inequalities used to describe and solve multi-step problems?

Common Core State Standards
Content Standards
7.EE.4, 7.EE.4a, 7.EE.4b, 8.EE.7, 8.EE.7a, 8.EE.7b
Mathematical Practices
1, 2, 3, 4, 7

Vocabulary
null or empty set
identity

What You'll Learn
- Solve multi-step equations.
- Solve multi-step inequalities.

Real-World Link
Field Trip A field trip is a fun and an exciting way to learn about something in a totally different way. Mr. Murphy's class of 20 students is going on a field trip to the science center. The total cost of admission and tickets for the 3-D movie for all of the students is $270.

Solve Multi-Step Equations

If x represents the science center's entry fee, the expression $20(x + 2.50)$ represents the total cost for Mr. Murphy's students. Suppose 15 of the students go on a field trip to an art museum where the entry fee is twice that of the science center's fee, and there is a fee of $1 for the audio tour. If the total cost for Mr. Murphy's students is the same at both museums, you can find the science center's entry fee by solving $20(x + 2.50) = 15(2x + 1)$.

Example 1

Tutor

Solve $20(x + 2.50) = 15(2x + 1)$. Check your solution.

$20(x + 2.50) = 15(2x + 1)$	Write the equation.
$20x + 50 = 30x + 15$	Distributive Property
$20x - 20x + 50 = 30x - 20x + 15$	Subtraction Property of Equality
$50 = 10x + 15$	Simplify.
$50 - 15 = 10x + 15 - 15$	Subtraction Property of Equality
$35 = 10x$	Simplify.
$3.5 = x$	Division Property of Equality; check your solution.

Got It? Do these problems to find out.

Solve each equation. Check your solution.

1a. $12m + 12 = 6(3m + 3)$

1b. $5(n - 3) = 3(n + 7)$

Some equations have *no* solution. When this occurs, the solution is the **null or empty set**, shown by the symbol Ø or {}. Other equations may have every number as their solution. An equation that is true for every value of the variable is called an **identity**.

Jason Edwards/National Geographic RF/Getty Images

Identities
An identity is an equation that shows that a number or expression is equivalent to itself.
Additive Identity
$a + 0 = a$
Multiplicative Identity
$a \cdot 1 = a$

Example 2

Tutor

Solve each equation.

a. $3(y - 5) + 25 = 3y + 10$

$3(y - 5) + 25 = 3y + 10$	Write the equation.
$3y - 15 + 25 = 3y + 10$	Use the Distributive Property.
$3y + 10 = 3y + 10$	Simplify.
$3y + 10 - 3y = 3y + 10 - 3y$	Subtraction Property of Equality
$10 = 10$	Simplify.

The statement $10 = 10$ is *always* true. The equation is an identity and the solution set is all numbers.

b. $-5s - 14 = 2(2s + 3) - 9s$

$-5s - 14 = 2(2s + 3) - 9s$	Write the equation.
$-5s - 14 = 4s + 6 - 9s$	Use the Distributive Property.
$-5s - 14 = 6 - 5s$	Simplify.
$-5s - 14 + 5s = 6 - 5s + 5s$	Addition Property of Equality
$-14 = 6$	Simplify.

The statement $-14 = 6$ is *never* true. The equation has no solutions and the solution set is $\emptyset$.

Got It? Do these problems to find out.

2a. $-2(3r + 4) = -5r - 8 - r$ **2b.** $14 + 8w = 4(8 + 2w)$

2a. $-2(3r+4) = -5r-8-r$
$-6r + (-8) = -6r - 8$
$-6r - 8 = -6r - 8$
INFINITE SOLUTIONS

2b $14 + 8w = 4(8+2w)$
$14 + 8w = 32 + 8w$
$-8w \quad -8w$
$14 \neq 32$
NO SOLUTION

Solve Multi-Step Inequalities

Solving a multi-step inequality is similar to solving a multi-step equation. As a first step, you can use the Distributive Property to remove grouping symbols.

Example 3

Tutor

Solve $4(x - 3) > 6$. Check your solution.

$4(x - 3) > 6$	Write the inequality.
$4x - 12 > 6$	Distributive Property
$+12 \quad +12$	Subtraction Property of Inequality
$4x > 18$	Simplify.
$\frac{4x}{4} > \frac{18}{4}$	Division Property of Inequality
$x > 4.5$	Simplify. Check your solution.

3a. $3 < 4(x+2)$
$3 < 4x + 8$
$-8 \quad -8$
$\frac{-5}{4} < \frac{4x}{4}$
$-\frac{5}{4} < x$
$x > -\frac{5}{4}$

3b. $4(b-3) \leq 72$
$4b - 12 \leq 72$
$+12 \quad +12$
$\frac{4b}{4} \leq \frac{84}{4}$
$b \leq 21$

Got It? Do these problems to find out.

Solve. Check your solution.

3a. $3 < 4(x + 2)$ **3b.** $4(b - 3) \leq 72$

Example 4

Mariella's parents have budgeted at most $575 for her Quinceañera celebration. The cost of the party room is $75. How much can the family spend per guest on food if each of the 40 guests receives a $5 favor?

First write an inequality to represent the situation.

Words	party cost ≥ room cost + the number of guests × the cost per guest
Variable	Let c = the food cost per guest so $c + 5$ = the total cost per guest.
Inequality	$575 \geq 75 + 40(c + 5)$

$575 \geq 75 + 40(c + 5)$	Write the Inequality.
$575 \geq 75 + 40c + 200$	Distributive Property
$575 \geq 40c + 275$	Simplify.
$575 - \mathbf{275} \geq 40c + 275 - \mathbf{275}$	Subtraction Property of Inequality
$300 \geq 40c$	Simplify.
$7.5 \geq c$	Division Property of Inequality

The family can spend at most $7.50 per guest on food.

Got It? Do this problem to find out.

4. Sofia recycled 3 pounds less than the amount that James recycled. Hannah recycled 3 times the amount that Sofia recycled. If they recycled a total of 53 pounds, how many pounds did Sofia recycle?

Watch Out!

Inequality Signs Do not reverse the inequality sign just because there is a negative sign in the inequality. Only reverse the sign when you multiply or divide by a negative number.

Example 5

Solve $5a - 8 \geq 4(a - 3)$. Graph the solution on a number line.

$5a - 8 \geq 4(a - 3)$	Write the inequality.
$5a - 8 \geq 4a - 12$	Distributive Property.
$5a - 8 - \mathbf{4a} \geq 4a - 12 - \mathbf{4a}$	Subtraction Property of Inequality
$a - 8 \geq -12$	Simplify.
$a - 8 + \mathbf{8} \geq -12 + \mathbf{8}$	Addition Property of Inequality
$a \geq -4$	Simplify.
	Graph the solution on a number line.

Got It? Do these problems to find out.

Solve. Graph each solution on a number line.

5a. $-2(k + 1) > -16 + 5k$

5b. $2p + 5 \geq 3(p - 6)$

Guided Practice

Solve. Check your solution. (Examples 1 and 2)

1. $4(x + 1) + 3 = 31$
2. $33 = 7(2p - 1) - 2$
3. $2(a - 2) = 3(a - 5)$
4. $16(z + 3) = 4(z + 9)$
5. $7(x + 2) = 2(x + 2)$
6. $3(d - 2) = 5(d + 8)$
7. $6x + 4 = 2(3x - 5)$
8. $20f + (-8f - 15) = 3(4f - 5)$
9. $3(1 + 2f) - 5 = 6f - 2$
10. $7x + 5 = 10(x - 7) - 3x$

Solve. Graph each solution on a number line. (Examples 3 and 5)

11. $-2(k - 2) \geq -20$
12. $(3r + 7)2 \leq -34$
13. $-2(g - 1) > g - 4$
14. $5p + 8 \geq 3(p + 6)$
15. $6(-2z + 5) < -19z + 16$
16. $10p \leq 7(2p - 4)$

17. You and three friends are going to the fair. The cost for parking is \$5 per car and admission to the fair is \$19 per person. If you have a total of \$113, what is the maximum amount each person can spend on food? (Example 4)

Independent Practice

Go online for Step-by-Step Solutions

Solve. Check your solution. (Examples 1 and 2)

18. $6n - 18 = 4(n + 2)$
19. $12y + 5(y - 6) = 4$
20. $12z + 4 = 2(5z + 8) - 12$
21. $d - 12 = 4(d - 6)$
22. $3x + 2 = 2(2x - 7)$
23. $6(y - 5) = 2(10 + 3y)$
24. $4(2c + 8) = 5(c + 4)$
25. $10 + 12p = 3(3 + 4p)$
26. $3x + 2 + 5(x - 1) = 8x + 17$
27. $10z + 4 = 2(5z + 8) - 12$

Solve. Graph each solution on a number line. (Examples 3 and 5)

28. $20 > 5(w + 3)$
29. $-32 \leq 9(3h + 2) + 4$
30. $3(6m - 4) \geq 24$
31. $10(3 + s) < 4s$
32. $3y - 6 > 4(y - 3)$
33. $8(2h + 6) \leq 12h + 20$
34. $3(3r + 5) \geq 24 + 10r$
35. $14t - 28 < 7(t + 6)$

31. $10(3+5) < 4s$
$30 + 10s < 4s$
$-10s \quad -10s$
$\frac{30}{-6} < \frac{-6s}{-6}$
$-5 > s$
$s < -5$
$10(3 + -6) < 4(-6)$
$10(-3) < -24$
$-30 < -24$

36. The perimeter of a rectangle is at least 50 centimeters. The length of the rectangle is one more than 3 times the width of the rectangle. What are the minimum dimensions of the rectangle? (Example 4)

37. **Financial Literacy** Tim is taking the train to Seattle to visit his grandparents. He has at most \$15.00 to spend on snacks and reading material. Granola bars cost \$1.15 each, and magazines cost \$1.25. If Tim buys the same number of granola bars and magazines, how many can he buy? (Example 4)

38. **Reason Abstractly** Sumi is considering two gyms. Gym A has a one-time membership fee of \$45 and costs \$18 per month. Gym B has a one-time membership fee of \$60 and costs \$18 per month. Write and solve an equation to find the number of months for which the two gyms have the same cost. Explain how your solution relates to the problem situation.

39. Nomar has earned scores of 73, 85, 91, and 82 on the first four of five math tests for the grading period. He would like to finish the grading period with a test average of at least 82. What score does Nomar need to earn on the fifth test in order to achieve his goal?

Solve.

40. $-0.2(3c + 15) = 3(0.8c - 8)$

41. $2(t + 12) - 6(2t - 3) = 14$

42. $5 - \frac{1}{2}(x - 6) < 4$

43. $6n - 18 \geq 4(n + 2.1)$

44. $2.01c - 6 = -0.15c + 6.96$

45. $\frac{1}{4}x + 13 > 0.25(2x - 32)$

46. $0.5(4x + 24) = 22x - 2$

47. $6(n + 2) + 3(3n - 5) = 57$

48. Cole is having his car repaired. The mechanic said it would cost at least \$375 for parts and labor. If the cost of the parts was \$150, and the mechanic charges \$60 an hour, how many hours is the mechanic planning to work on the car?

49 A good rule to know when training for a marathon is that you will generally have enough endurance to finish a race that is 3 times your average daily distance. Tammy wants to be able to run *at least* the standard marathon distance of 26.2 miles. The length of her current daily run is about 4 miles. To the nearest hundredth, by how many miles should she increase her daily run to meet her goal?

Solve. Justify each step in the solution. Use a Property of Equality or Inequality when necessary.

50. $4(y - 3) = 2(3y + 10)$

51. $5(2f - 1) = 3(f + 3)$

52. $-1.2(w + 1.1) \leq 6.18$

53. $p > \frac{2}{3}\left(p - \frac{1}{2}\right)$

H.O.T. Problems Higher Order Thinking

54. **Identify Structure** Write a multi-step inequality that can be solved by first adding 3 to each side.

55. **Identify Structure** Explain how you can solve $45 > -6x + 3$ without multiplying or dividing by a negative number.

56. **Find the Error** Jada is solving $3x - 9 \leq 5(x + 10)$. Find her mistake and correct it.

$$3x - 9 \leq 5(x + 10)$$
$$3x - 9 \leq 5x + 10$$
$$-2x \leq 19$$
$$x \geq -9.5$$

57. **Persevere with Problems** Use the information in Example 2 about equations that have no solutions or those that are identities to solve the following inequalities. Justify each step in the solution.

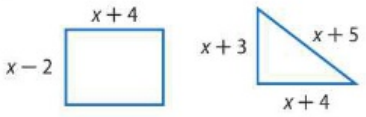
a. $5x - 6 \geq 3(x - 2) + 2x$

b. $12p + 17 \leq 3(4p - 8)$

58. **Building on the Essential Question** Explain how to determine if an equation has no solution, one solution, or if all numbers are solutions. Use examples with your explanation.

Standardized Test Practice

59. Find the value of x so that the polygons have the same perimeter.



A 3 **C** 8
B 6 **D** 12

60. **Short Response** Damon can spend at most \$150 on supplies for a school party. He has already spent \$78.80 on food and \$29.95 on decorations. He plans to buy cases of juice that cost \$8.25 per case. Write and solve an inequality to find the number of cases of juice Damon can buy.

61. What is the solution of this inequality?

$$-4x + 16 \geq -4$$

F $x \geq 3$
G $x \leq 3$
H $x \geq 5$
J $x \leq 5$

62. Sandra's scores on the first five science tests are shown. Which inequality represents the score she must receive on the sixth test to have an average score of more than 88?

Test	Score
1	85
2	84
3	90
4	95
5	88

A $s \geq 86$ **C** $s < 88$
B $s \leq 88$ **D** $s > 86$

Common Core Review

Solve each inequality. Graph each solution on a number line. 7.EE.4

63. $-25t \leq 400$ **64.** $8 > \frac{q}{3}$ **65.** $14 \geq 7 + a$

66. $-13 \geq x - 8$ **67.** $-\frac{3}{4} < w - 1$ **68.** $3 \leq \frac{1}{2} + a$

Write an inequality for each sentence. 7.EE.4

69. Kyle's earnings were no more than \$60.

70. The 10 kilometer race time of 86 minutes was greater than the winner's time.

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal. 7.NS.2d

71. $\frac{1}{5}$ **72.** $-\frac{5}{8}$ **73.** $7\frac{3}{10}$

74. $\frac{1}{9}$ **75.** $-3\frac{3}{4}$ **76.** $-\frac{5}{11}$

77. **Financial Literacy** The formula $P = I - E$ is used to find the profit P when income I and expenses E are known. One month a small business has an income of \$19,592 and expenses of \$20,345. 7.NS.1

a. What is the profit for the month?

b. What does a negative profit mean?

78. **STEM** The diameter of Earth is approximately 1.28×10^4 kilometers. (8.EE.4)

a. The diameter of Jupiter is about 11.1 times the diameter of Earth. Write the diameter of Jupiter in standard notation.

b. Write the diameter of Jupiter in scientific notation.

Divide. 6.NS.3

79. $7.2 \div 2$ **80.** $\$3.75 \div 5$ **81.** $\$25.90 \div 3.5$ **82.** $29.14 \div 4.7$

$6(n+2)+3(3n-5)=57$

$6(n)+6(2)+3(3n)-3(5)$

$6n+12+9n+-15=57$

$-6n \qquad -6n$

$6(n+2)=3(3n-5)=57$

$6(n)+6(2)=3(3n)-3(5)$

$6n+12=9n+-15 \quad 57$

$-6n \qquad -6n$

$\frac{1}{4}x+13>0.25(2x-32) \longrightarrow$

$\frac{1}{4}(83)+13>\frac{1}{4}(2\cdot 83-32)$

$\frac{1}{4}x+13>\frac{1}{4}(2x-32)$

$\frac{1}{4}x=\frac{x}{4}$

$\frac{1}{4}x+13>\frac{1}{2}x-8$

$-\frac{1}{4}x \qquad -\frac{1}{4}x$

$13>\frac{1}{4}x-8$

$+8 \qquad +8$

$4\cdot 21>\frac{1}{4}x\cdot\frac{4}{1}$

$84>x$

$x<84$

1a. $12m + 12 = 6(3m + 3)$

$12m + 12 = 18m + 18$

$-12m \qquad -12m$

$12 = 6m + 18$

$-18 \qquad -18$

$\frac{-6}{6} = \frac{6m}{6}$

$-1 = m$

$12(-1) + 12 = 6(3 \cdot -1 + 3)$

$-12 + 12 = 6(-3 + 3)$

$0 = 6(0)$

$0 = 0$

1b. $5(n - 3) = 3(n + 7)$

$5n - 15 = 3n + 21$

$-3n \qquad -3n$

$2n - 15 = 21$

$+15 \qquad +15$

$\frac{2n}{2} = \frac{36}{2}$

$n = 18$

19 $12y + 5(y - 6) = 4$

$12y + 5y - 30 = 4$

$17y - 30 = 4$

$+30 \quad +30$

$\frac{17y}{17} = \frac{34}{17}$

$y = 2$

$12(2) + 5(2-6) = 4$

$24 + 5(-4) = 4$

$24 + -20 = 4$

$4 = 4$

23. $6(y-5) \neq 2(10+3y)$

$6y - 30 \neq 20 + 6y$

$-6y \qquad -6y$

$-30 = 20$

$6y - 30$

$6y + 20$

$6y + 40 = 40 + 6y$

$-6y \qquad -6y$

$40 = 40$

$3y + 20 = 6y + 40$

27. $10z + 4 \stackrel{?}{=} 2(5z + 8) - 12$

$10z + 4 = 10z + 16 - 12$

$10z + 4 = 10z + 4$

37. Financial Literacy Tim is taking the train to Seattle to visit his grandparents. He has at most \$15.00 to spend on snacks and reading material. Granola bars cost \$1.15 each, and magazines cost \$1.25. If Tim buys the same number of granola bars and magazines, how many can he buy? (Example 4)

x = GB MAG

$$1.25x + 1.15x \leq 15$$

$$\frac{2.40x}{2.40} \leq \frac{15}{2.40}$$

$$x \leq 6.25$$

45. $\frac{1}{4}x + 13 \geq 0.25(2x - 32)$

$\frac{1}{4}x + 13 \geq 0.5x - 8$

$+8 \quad +8$

$\frac{1}{4}x + 21 \geq 0.5x$

$-0.25x$

$-\frac{1}{4}x$

$21 \geq 0.25x$

$\frac{21}{0.25} \quad \frac{0.25x}{0.25}$

$84 \geq x$

$x \leq 84$

$4(21) \geq 4(\frac{1}{4})x$

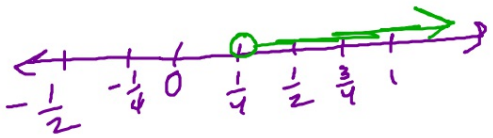
$84 \geq x$

67. $-\frac{3}{4} \leq w - 1$

$+1 \qquad +1$

$\frac{1}{4} \leq w$ ←

$w \geq \frac{1}{4}$



$\frac{1}{4}\left(2x - \frac{3}{5}y\right) >$